HVDC PLL

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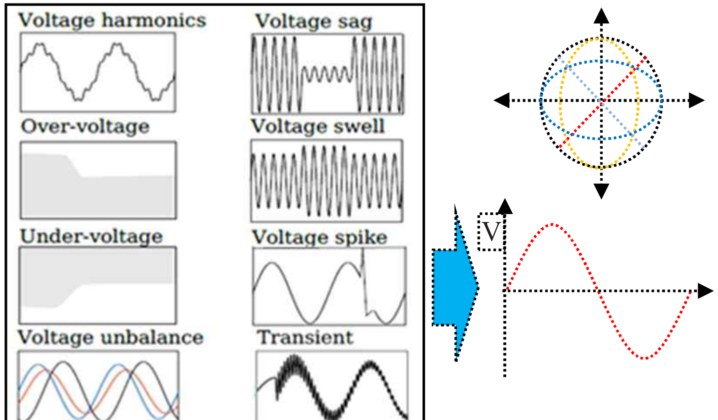
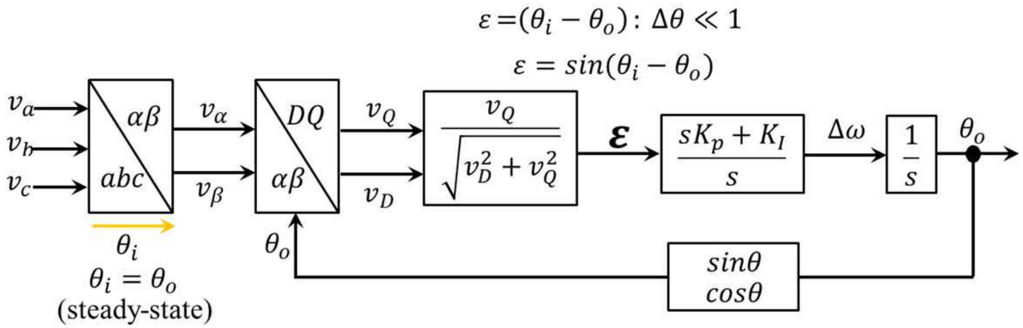
Purposes of VSC HVDC PLL

“ ***To control the HVDC converter according to the phase of AC network*** ”



Exact Phase!



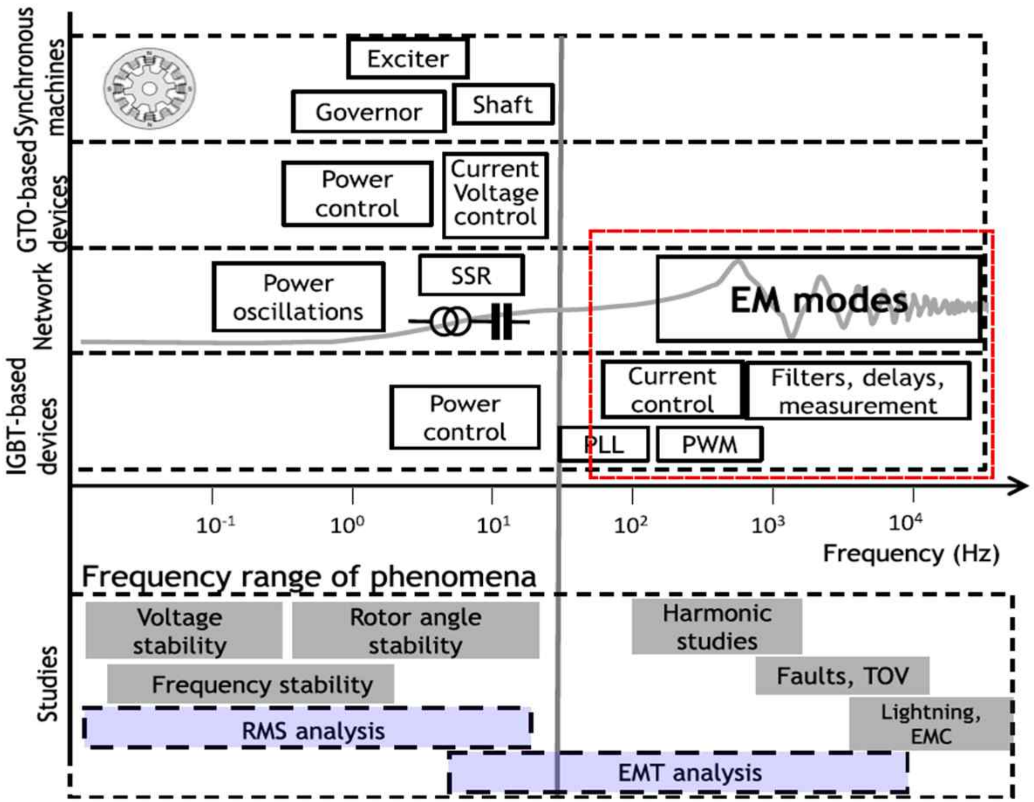
State Condition Control

- Gain Tuning/Control Zero
- Freezing/Disable/Mode
- Blocking
- Command/Limiter Changing
- Stability Up

$$\textbf{Stability} = \begin{cases} \theta_i : \text{by power system} \\ \frac{X_g}{R_g}, \frac{1}{sJ_g}, \frac{Q_g}{P_g}, Z_g + \frac{Z_1}{Z_2} \\ \theta_o : \text{by PLL dynamic} \\ \frac{s^2 K_D}{s^2 + 2\zeta\omega_n s + \omega_n^2} \end{cases}$$

$$\boldsymbol{G_{PLL}} = \frac{\Delta\theta_o}{\Delta\theta_i} = \frac{\boldsymbol{V}K_p s + \boldsymbol{V}K_I}{s^2 + \boldsymbol{V}K_p s + \boldsymbol{V}K_I}$$

	Synchronous Machine	Converter PLL
Theta (Angle)	Controlled	Controlling
Stability of AC network	Passive	Active
Operation	With inertia	No inertia
Temporary Blocking	Impossible	Possible
Weak Load Operation	Impossible	Possible
Operation **Range(Hz)**	> 0.01 (AVR)	> 100



PLL Requirements

▶ Among controller functions
- Firstly, Active state
- Start Up

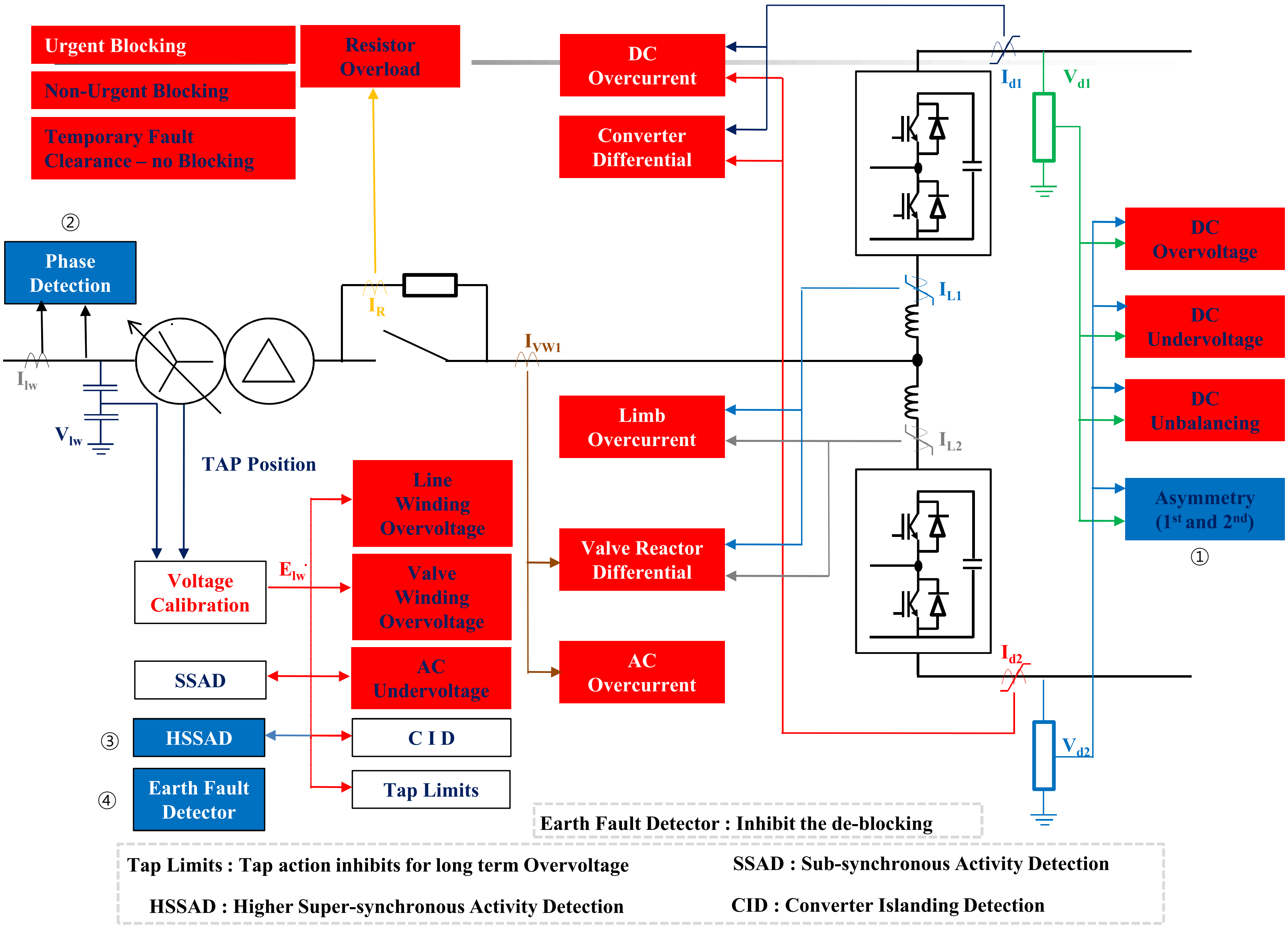
▶ Control or Idling

▶ PLL Temporary Blocking
- FRT(Fault Ride Through)
- Transient Stability
- Sag/Swell, Voltage Stability
- 1-phase fault/3-phase fault
- Islanding
- Singularity Instability

▶ Blocking or Freezing

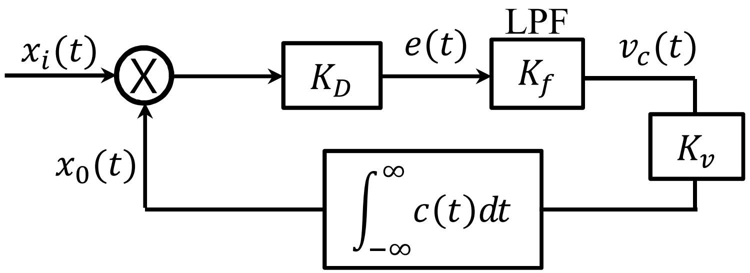
▶ PLL Robust
- Harmonic Stability
- TOV
- Pos./Neg. Sequence
- Super-synchronous Stability
- Sub-synchronous Resonance

▶ PLL Monitoring
- Theta and Voltage → AC network
- Control Freezing or Trip


Urgent Blocking
Non-Urgent Blocking
Temporary Fault Clearance – no Blocking
Resistor Overload
DC Overcurrent
Converter Differential
②
Phase Detection
I_R
I_{d1}
V_{d1}
I_{L1}
I_{VW1}
I_{lw}
V_{lw}
TAP Position
DC Overvoltage
DC Undervoltage
DC Unbalancing
Asymmetry (1st and 2nd)
①
Limb Overcurrent
I_{L2}
Line Winding Overvoltage
Valve Winding Overvoltage
Valve Reactor Differential
Voltage Calibration
E_{lw}
SSAD
AC Undervoltage
AC Overcurrent
I_{d2}
③
HSSAD
C I D
V_{d2}
④
Earth Fault Detector
Tap Limits
Earth Fault Detector : Inhibit the de-blocking
Tap Limits : Tap action inhibits for long term Overvoltage
SSAD : Sub-synchronous Activity Detection
HSSAD : Higher Super-synchronous Activity Detection
CID : Converter Islanding Detection

Phase Locked Loop(PLL)

$$\frac{1}{2}[\cos((\omega_i-\omega_o)t+(\theta_i-\theta_o))+\cos((\omega_i+\omega_o)t+(\theta_i+\theta_o))]$$



$$x_i(x)=A\cos(\omega_i+\theta_i)$$

$$x_0(x)=B\cos(\omega_o+\theta_o)$$

$$PLL=x_i(x)\otimes x_o(x)$$

$$A\cos(\omega_i+\theta_i)\times B\cos(\omega_o+\theta_o)$$

$$e(t)=K_D[\cos((\omega_i-\omega_o)t+(\theta_i-\theta_o))+\cos((\omega_i+\omega_o)t+(\theta_i+\theta_o))]$$

$$v_c(t)=K_D\cos((\omega_i-\omega_o)t+(\theta_i-\theta_o))$$ Gain : K_D, Low pass filter : 2 x ω_i

$$x_0(t)=B\sin(\omega_i+\boldsymbol{\varphi_o})\quad \theta_0=(\omega_i-\omega_o)t+\varphi_0\quad v_c(t)=K_D\cos((\theta_i-\varphi_o))$$: **DC term**

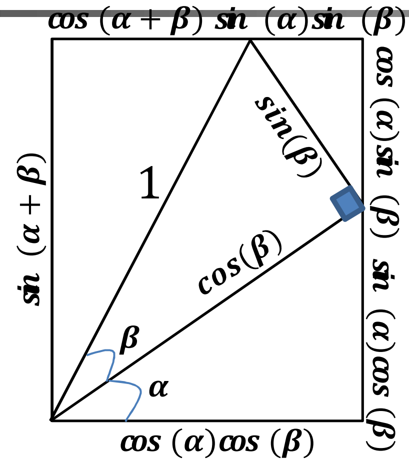
$$\omega_{inst}=\frac{d}{dt}(\omega_o t+\theta_o)=\omega_0+\frac{d\theta_0}{dt}\qquad \frac{d\theta_0}{dt}=K_v\cdot V_c(t)\qquad \omega_i-\omega_o=K_DK_v\cos(\theta_i-\varphi_o)$$

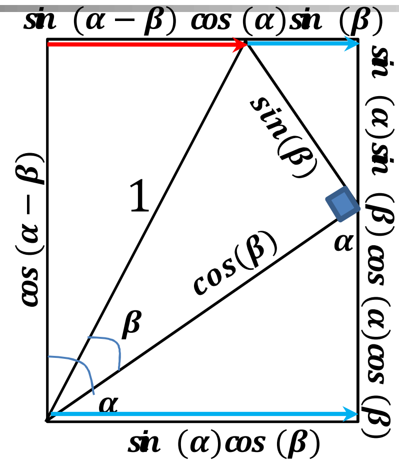
$$\varphi_0=\theta_i-\cos^{-1}\left(\frac{\omega_i-\omega_o}{K_v\cdot K_D}\right)\qquad v_c=\frac{\omega_i-\omega_0}{K_v}\qquad \omega_{inst}=\omega_0+K_vK_D=\omega_i$$

$$\theta_2=\varphi_o+\frac{\pi}{2}\qquad v_c(t)=K_D\sin((\theta_i-\theta_2))\qquad v_c(t)=K_D(\theta_i-\theta_2)$$

$$\varphi_0=\theta_i-\cos^{-1}\left(\frac{\omega_i-\omega_o}{K_v\cdot K_D}\right)$$

Phase Locked Loop(PLL)





$$\cos(\alpha-\beta) = \sin\alpha\cdot\sin\beta + \cos\alpha\cdot\cos\beta$$

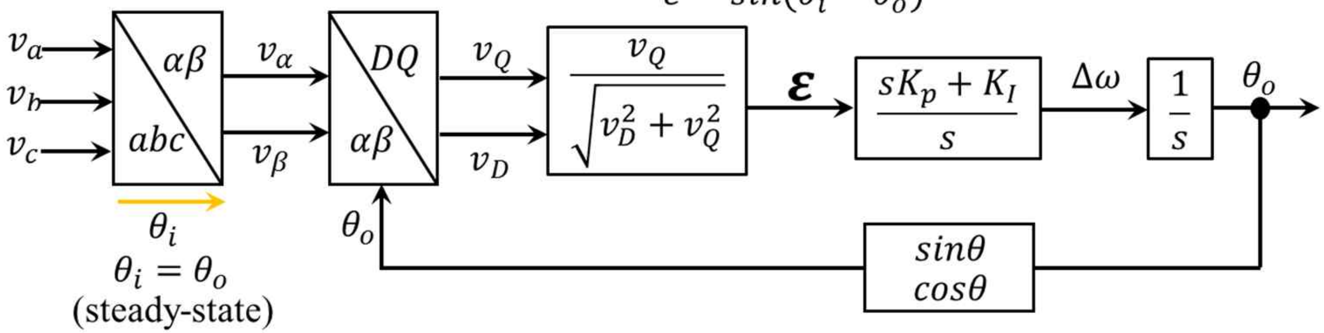
$$\sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta$$

$$v_Q = -V\cos\alpha\cdot\sin\beta + V\sin\alpha\cdot\cos\beta = \boldsymbol{V\sin(\alpha-\beta)}$$

$$v_D = V\cos\alpha\cdot\cos\beta + V\sin\alpha\cdot\sin\beta = V\cos(\alpha-\beta)$$

$\varepsilon = (\theta_i - \theta_o)$: $\Delta\theta \ll 1$

$\varepsilon = sin(\theta_i - \theta_o)$



$$\sin(\Delta\theta) = \Delta\theta$$

$$\frac{\sin\theta - \theta}{\sin\theta} < \varepsilon(\%)$$

$$\begin{cases} 1\% : \pm 13.98^o \\ 0.1\% : \pm 4.4^o \end{cases}$$

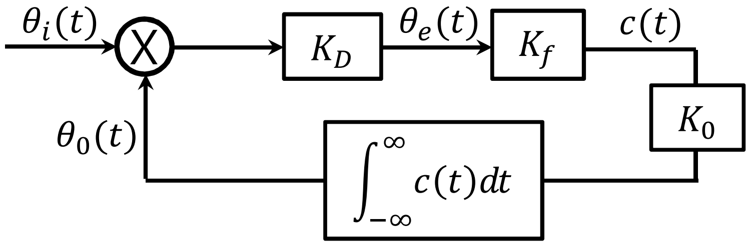
$$v_Q(t) = -V\cos(\omega_i t + \theta_i)\cdot\sin(\omega_o t + \varphi_o) + V\sin(\omega_i t + \theta_i)\cdot\cos(\omega_o t + \varphi_o)$$

$$v_Q(t) = -\frac{V}{2}\sin\left((\omega_i+\omega_o)t + \theta_i + \varphi_o\right) + \boldsymbol{\frac{V}{2}\sin\left((\omega_i-\omega_o)t + \theta_i - \varphi_o\right)}$$

$$+\frac{V}{2}\sin\left((\omega_i+\omega_o)t + \theta_i + \varphi_o\right) + \boldsymbol{\frac{V}{2}\sin\left((\omega_i-\omega_o)t + \theta_i - \varphi_o\right)}$$

$$\begin{cases} 2\times frequency \quad removing \\ AC\ term \rightarrow DC\ term \end{cases}$$

Phase Locked Loop(PLL)-1st order loop



$$c(t) = K_f \cdot \theta_e\ (t)$$

$$\theta_e(t) = K_D(\theta_i(t) - \theta_o(t))$$

$$= K_D(\theta_i(t) - K_0 \int c(t)dt)$$

$$\theta_e(t) = K_D(\theta_i(t) - K_0 \int K_f \cdot \theta_e(t)\ dt)$$

$$\theta_e'(t) + K_D K_f K_o \theta_e(t) = K_D \theta_i'(t)$$

$$\theta_e(t) = e^{-(K_D K_f K_0 t)} \int (e^{K_D K_f K_0 t})(\theta_i'(t))\mathrm{dt} + ce^{-(K_D K_f K_0 t)}$$

$$f' + \alpha f = g$$

$$f = e^{-A}\left(\int g e^{A}\right) + \text{æ}^{-A}$$

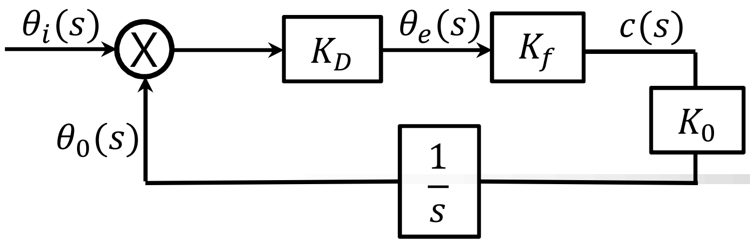
$$A(t) = \int \alpha(t)dt = K_D K_f K_0 t$$

$\theta_i(t) = \Delta\theta$ $\quad \theta_i'(t) = 0 \quad \theta_{e0}(0) = \Delta\theta$

Case 1
Phase Step up

$$\theta_e(t) = \Delta\theta e^{-(K_D K_f K_0 t)}$$

$$\lim_{t\to\infty}\{\Delta\theta e^{-(K_D K_f K_0 t)}\} = \mathbf{0}$$



$$\theta_e = H_e \cdot \theta_i \quad H_e(s) = \frac{K_D s}{s + K_o K_D K_f}$$

$$\theta_e(s) = \frac{\boldsymbol{\Delta\theta}}{\boldsymbol{s}} \cdot \frac{K_D s}{s + K_o K_D K_f}$$

$$\theta_e(t) = K_D \cdot e^{-(K_D K_f K_0 t)} \cdot u(t)$$

Phase Locked Loop(PLL)-1st order loop

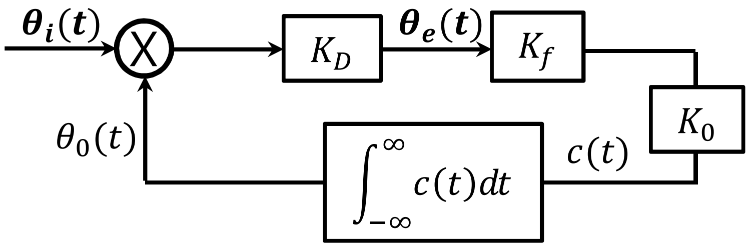
Case 2
Frequency Step-up

$$\theta_i(t) = 2\pi\Delta f\mathrm{t} \quad \theta_i'(t) = \frac{d}{dt}\{2\pi\Delta f\mathrm{t}\} = 2\pi\Delta f \quad \theta_i(s) = \frac{2\pi\Delta f}{s^2}$$

$$\theta_e(s) = \frac{2\pi\Delta f}{s^2}\cdot\frac{K_D s}{s + K_o K_D K_f} \quad \theta_e(t) = \frac{K_D\cdot 2\pi\Delta f}{K_0 K_D K_f}\cdot\left[1 - e^{-K_D K_0 K_f t}\right]\cdot u(t)$$

$$\lim_{t\to\infty}\{\boldsymbol{\frac{2\pi\Delta f}{K_D K_f K_0}} - \frac{2\pi\Delta f}{K_D K_f K_0}e^{-(K_D K_f K_0 t)}\} = \boldsymbol{\frac{2\pi\Delta f}{K_D K_f K_0}}$$

Phase Locked Loop(PLL)-1st order loop



$$H_e(s) = \frac{\theta_e}{\theta_i} \qquad H_0(s) = \frac{\theta_0}{\theta_i}$$

$$H_0(s) = \frac{G_{PLL}}{s + G_{PLL}} = \frac{[K_D K_f K_0]}{s + [K_D K_f K_0]}$$

$$H_e(s) = \frac{\theta_e}{\theta_i} = \frac{Forward \quad Gain}{1 + Total \quad Gain} = \frac{sK_D}{s + [K_D K_f K_0]}$$

$$\theta_e(s) = \theta_i(s) H_e(s)$$

$$\theta_i(s) = \frac{\Delta\theta}{s}$$ (Phase Jump)

$$\theta_i(s) = \frac{2\pi\Delta f}{s^2}$$ (Frequency Jump)

$$\theta_i(s) = 2 \cdot \frac{2\pi \dot{f}}{s^3}$$ (Frequency Ramp)

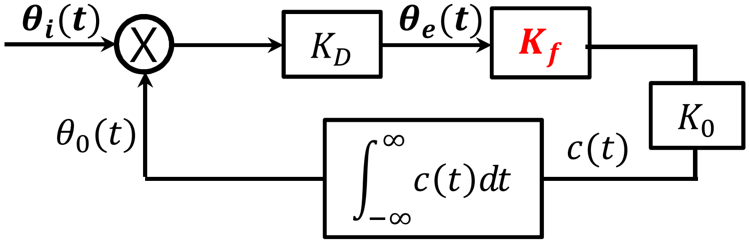
$$\theta_e(s) = \frac{\Delta\theta}{s} \cdot \frac{sK_D}{s + [K_D K_f K_0]} \qquad \theta_e(t) = K_D \Delta\theta_e \cdot (e^{-K_D K_o K_f t}) \cdot u(t)$$

$$\theta_e(s) = \frac{2\pi\Delta f}{s^2} \cdot \frac{sK_D}{s + [K_D K_f K_0]} \qquad \theta_e(t) = \frac{K_D 2\pi\Delta f}{K_D K_o K_f} \cdot (1 - e^{-(K_D K_o K_f t)}) \cdot u(t)$$

$$\theta_e(s) = 2 \cdot \frac{2\pi \dot{f}}{s^3} \cdot \frac{sK_D}{s + [K_D K_f K_0]}$$

$$\theta_e(t) = \frac{K_D 4\pi \dot{f}}{[K_D K_o K_f]^2} \cdot \left(t\left(K_D K_o K_f\right) + e^{-(K_D K_o K_f t)} - 1\right) \cdot u(t)$$

Phase Locked Loop(PLL) - 2nd order loop



$$H_0(s) = \frac{G_{PLL}}{s + G_{PLL}} = \frac{[K_D \boldsymbol{K_f} K_0]}{s + [K_D \boldsymbol{K_f} K_0]}$$

$$H_e(s) = \frac{sK_D}{s + [K_D \boldsymbol{K_f} K_0]}$$

$$F_{fl}(s) = \frac{\mathbf{1}}{\mathbf{1} + \boldsymbol{\tau_1 s}}$$

(lag filter)

$$F_{fa2}(s) = \frac{\mathbf{1} + \boldsymbol{\tau_2 s}}{\mathbf{1} + \boldsymbol{\tau_1 s}}$$

(lead-lag filter)

$$F_{fa2}(s) = \frac{\boldsymbol{\tau_2 s} + \mathbf{1}}{\boldsymbol{\tau_1 s}}$$

(PI filter)

$$H_0(s) = \frac{K_D K_0 \left[\frac{\mathbf{1}}{\mathbf{1} + \boldsymbol{\tau_1 s}}\right]}{s + K_D K_0 \left[\frac{\mathbf{1}}{\mathbf{1} + \boldsymbol{\tau_1 s}}\right]} = \frac{K_D K_o}{\tau_1 s^2 + s + K_D K_o} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n = \sqrt{\frac{K_D K_o}{\tau_1}} \qquad \zeta = \frac{1}{2\tau_1 \omega_n} \qquad H_e(s) = \frac{K_D(s^2 + 2\zeta\omega_n s)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$H_0(s) = \frac{K_D K_0 \left[\frac{\mathbf{1} + \boldsymbol{\tau_2 s}}{\mathbf{1} + \boldsymbol{\tau_1 s}}\right]}{s + K_D K_0 \left[\frac{\mathbf{1} + \boldsymbol{\tau_2 s}}{\mathbf{1} + \boldsymbol{\tau_1 s}}\right]} = \frac{K_D K_o \tau_2 s + K_o K_D}{\tau_1 s^2 + s(1 + K_D K_o \tau_2) + K_o K_D}$$

$$\omega_n = \sqrt{\frac{K_D K_o}{\tau_1}} \qquad \zeta = \frac{1 + K_o K_D \tau_2}{2\sqrt{K_o K_D \tau_1}} \qquad H_e(s) = \frac{s^2 \tau_1 \omega_n^2 / K_o + s\omega_n^2 / K_o}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Phase Locked Loop(PLL) - PI controller

$$G_{PLL}(s) = \frac{K_p s + K_I}{s} = \frac{\frac{K_P}{K_I}s + 1}{\frac{1}{K_I}s} = \frac{\tau_2 s + 1}{\tau_1 s}$$

$$H_0(s) = \frac{G_{PLL}}{s + G_{PLL}} = \frac{K_D K_0 \left[\frac{\tau_2 s + 1}{\tau_1 s}\right]}{s + K_D K_0 \left[\frac{\tau_2 s + 1}{\tau_1 s}\right]} = \frac{K_D K_o[\tau_2 s + 1]}{\tau_1 s^2 + K_D K_o[\tau_2 s + 1]}$$

$$H_0(s) = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \omega_n = \sqrt{\frac{K_D K_o}{\tau_1}} \quad \zeta = \frac{\tau_2 \omega_n}{2} \quad H_e(s) = \frac{s^2 K_D}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\theta_e(s) = \frac{\Delta\theta}{s} \cdot \frac{K_D s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \lim_{s\to 0}[s\frac{\Delta\theta}{s} \cdot \frac{K_D s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}] = 0$$

$$\theta_e(t) = \Delta\theta \cdot K_D \cdot e^{-\zeta\omega_n t} z(t)$$

Case 1
Phase Step up

$$z(t) = \begin{cases} \left(\cos\left(\omega_n\sqrt{1-\zeta^2 t}\right) - \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\left(\omega_n\sqrt{1-\zeta^2 t}\right)\right) : \zeta < 1 \\ (1 - \omega_n t) : \zeta = 1 \\ \left(\cosh\left(\omega_n\sqrt{\zeta^2 - 1t}\right) - \frac{\zeta}{\sqrt{1-\zeta^2}}\sinh\left(\omega_n\sqrt{\zeta^2 - 1t}\right)\right) : \zeta > 1 \end{cases}$$

Phase Locked Loop (PLL) - PI controller

$$\theta_e(s) = \frac{2\pi\Delta f}{s^2} \cdot \frac{K_D s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad \lim_{s\to 0}[s\frac{2\pi\Delta f}{s^2} \cdot \frac{K_D s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}] = 0$$

Case 2
Frequency Step up

$$\theta_e(t) = \frac{2\pi\Delta f}{\omega_n} \cdot e^{-\zeta\omega_n t} \cdot z_2\,(t)$$

$$z_2(t) = \begin{cases} \dfrac{\zeta}{\sqrt{1-\zeta^2}} \sin\left(\omega_n\sqrt{1-\zeta^2 t}\right) & : \zeta < 1 \\ (\omega_n t) \ : \ \zeta = 1 \\ \dfrac{\zeta}{\sqrt{1-\zeta^2}} \sinh\ (\omega_n\sqrt{\zeta^2 - 1t}) & : \zeta > 1 \end{cases}$$

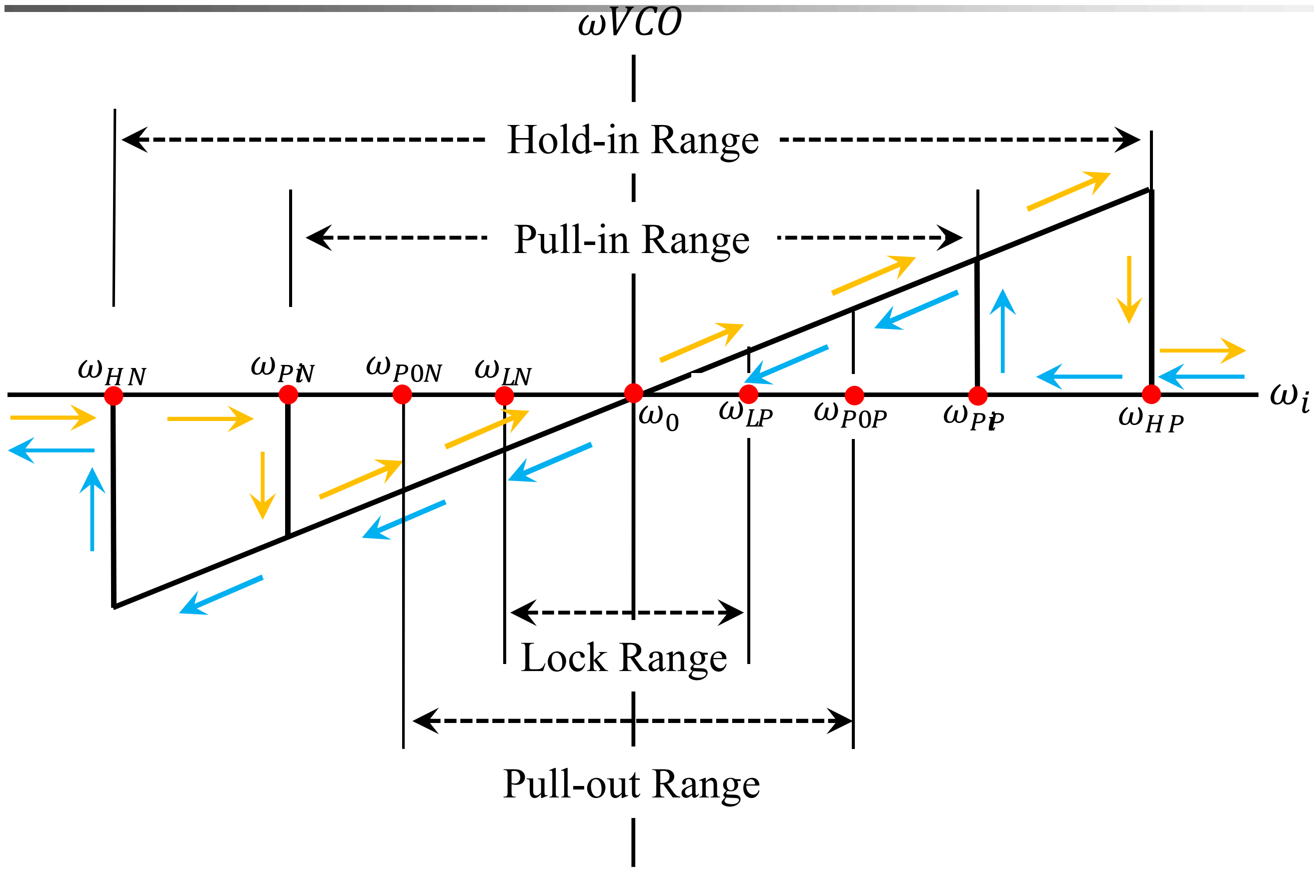
$$\theta_e(s) = 2 \cdot \frac{2\pi\dot{f}}{s^3} \cdot \frac{K_D s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \lim_{s\to 0}[s \cdot \frac{4\pi\dot{f}}{s^3} \cdot \frac{K_D s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}] = \boldsymbol{\frac{K_D \cdot 4\pi\dot{f}}{\omega_n^2}}$$

Case 3
Frequency Ramp

$$\theta_e(t) = \frac{4\pi\dot{f}}{\omega_n^2}(1 - e^{-\zeta\omega_n t}) \cdot z_3\,(t)$$

$$z_3(t) = \begin{cases} (\cos\ \left(\omega_n\sqrt{1-\zeta^2 t}\right) + \dfrac{\zeta}{\sqrt{1-\zeta^2}} \sin\ (\omega_n\sqrt{1-\zeta^2 t})) : \zeta < 1 \\ (\omega_n t) \ : \ \zeta = 1 \\ (\cosh\ \left(\omega_n\sqrt{\zeta^2 - 1t}\right) + \dfrac{\zeta}{\sqrt{1-\zeta^2}} \sinh\ (\omega_n\sqrt{\zeta^2 - 1t})) : \ \zeta > 1 \end{cases}$$

Phase Locked Loop (PLL)



$\omega_{HN} = \omega_0 - \Delta\omega_H$

$\omega_{PiN} = \omega_0 - \Delta\omega_{Pi}$

$\omega_{PoN} = \omega_0 - \Delta\omega_{Po}$

$\omega_{LN} = \omega_0 - \Delta\omega_L$

$\omega_{HP} = \omega_0 + \Delta\omega_H$

$\omega_{PiP} = \omega_0 + \Delta\omega_{Pi}$

$\omega_{PoP} = \omega_0 + \Delta\omega_{Po}$

$\omega_{LP} = \omega_0 + \Delta\omega_L$

$\Delta\omega_H$ Hold range : the frequency range over which an PLL can maintain phase tracking

$\Delta\omega_{Pi}$ Pull-in range : the range within which an PLL will always become locked

$\Delta\omega_{Po}$ Pull-out range : the dynamic limit for stable operation of an PLL

$\Delta\omega_L$ Lock range : the frequency range within which a PLL locks within one single beat note

Phase Locked Loop(PLL)

$\Delta\omega_H$ **Hold Range** : Frequency offset of input that causes a phase error of $\pm\frac{\pi}{2}$

$$\omega_1 = \omega_0 \pm \Delta\omega_H \qquad \theta_1(t) = \pm\Delta\omega_H t \qquad \theta_1(s) = \frac{\Delta\omega}{s^2}$$

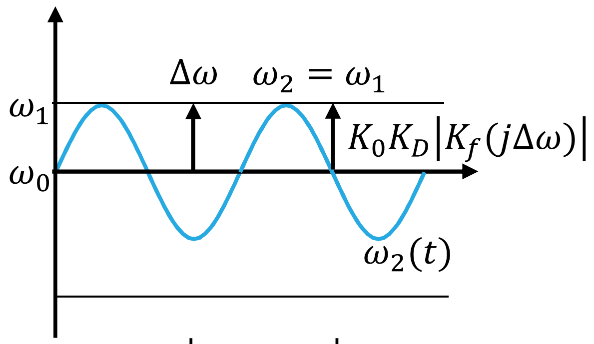
$$\theta_e(s) = \frac{sK_D}{s + [K_D K_f K_0(s)]}\cdot\frac{\Delta\omega}{s^2} \qquad \lim_{s\to 0}\{s\cdot\frac{sK_D}{s + [K_D K_f K_0(s)]}\cdot\frac{\Delta\omega}{s^2}\} = \frac{\Delta\omega}{K_D K_0 K_f(0)}$$

$$\lim_{t\to\infty} \sin\ \theta_e(t) = \frac{\Delta\omega_H}{K_0 K_D K_f(0)} \qquad \Delta\omega_H = \pm K_0 K_D K_f\ (\theta_e = \pm\frac{\pi}{2})$$

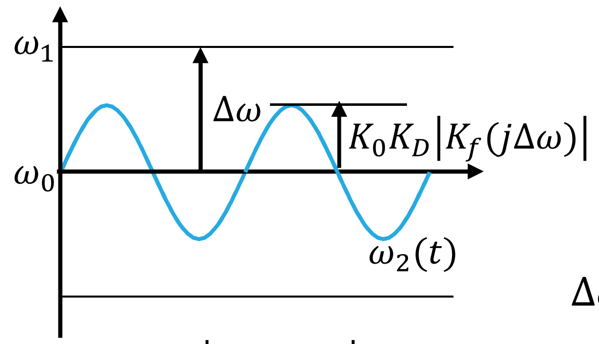
$$\Delta\omega_H = \pm K_0 K_D \qquad \Delta\omega_H = \pm K_a K_0 K_D \qquad \Delta\omega_H = \infty$$

(lag filter) (lead-lag filter) (PI filter)

$\Delta\omega_L$ **Lock Range** : Locks within one single beat note



$K_0 K_D|K_f(j\Delta\omega)| \geq \Delta\omega$



$K_0 K_D|K_f(j\Delta\omega)| \leq \Delta\omega$

$$\Delta\omega_L \begin{cases} \pm K_0 K_D \frac{\tau_2}{\tau_1} : \text{lag filter} \\ \pm K_a \frac{\tau_2}{\tau_1} : \text{lead lag filter} \\ \pm \frac{\tau_2}{\tau_1} : \text{PI filter} \end{cases}$$

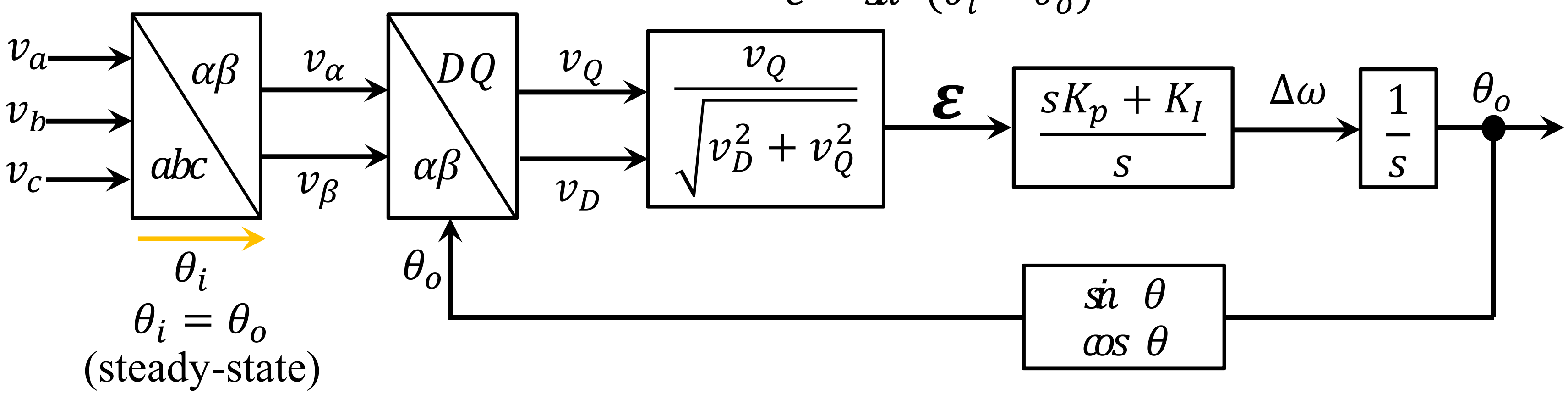
$$\Delta\omega_L = \pm 2\zeta\omega_n \qquad T_L = \frac{1}{f_n} = \frac{2\pi}{\omega_n}$$

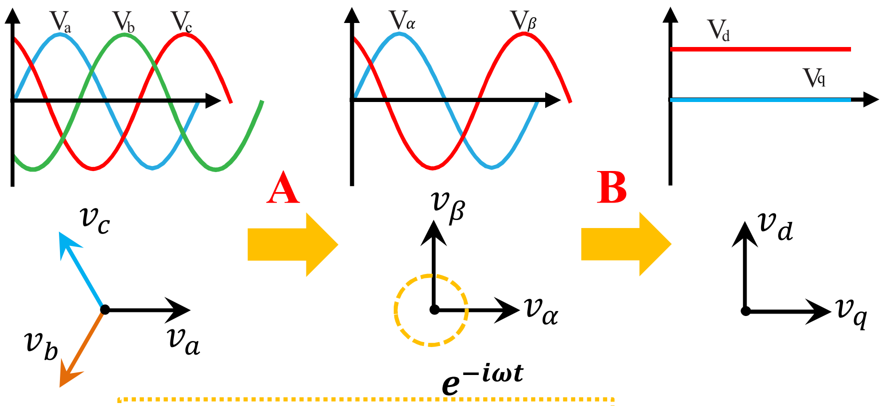
Lock-in time : one cycle

3 Phase Locked Loop (PLL)

$$\varepsilon = (\theta_i - \theta_o):\ \Delta\theta \ll 1$$

$$\varepsilon = \sin(\theta_i - \theta_o)$$





$$v_Q = -v_\alpha \sin(\theta_o) + v_\beta \cos(\theta_o)$$
$$= -V\cos(\theta_i)\sin(\theta_o) + V\sin(\theta_i)\cos(\theta_o)$$
$$= V\sin(\theta_i - \theta_o)$$

$$v_D = v_\alpha \cos(\theta_o) + v_\beta \sin(\theta_o)$$
$$= V\cos(\theta_i)\cos(\theta_o) + V\sin(\theta_i)\sin(\theta_o)$$
$$= V\cos(\theta_i - \theta_o)$$

$$\begin{cases} \theta_i : \text{by power system} \\ \dfrac{X_g}{R_g}, \dfrac{1}{SJ_g}, \dfrac{Q_g}{P_g}, Z_g + \dfrac{Z_1}{Z_2} \\ \theta_o : \text{by PLL dynamic} \\ \dfrac{s^2 K_D}{s^2 + 2\zeta\omega_n s + \omega_n^2} \end{cases}$$

v_Q and v_D : **DC term**

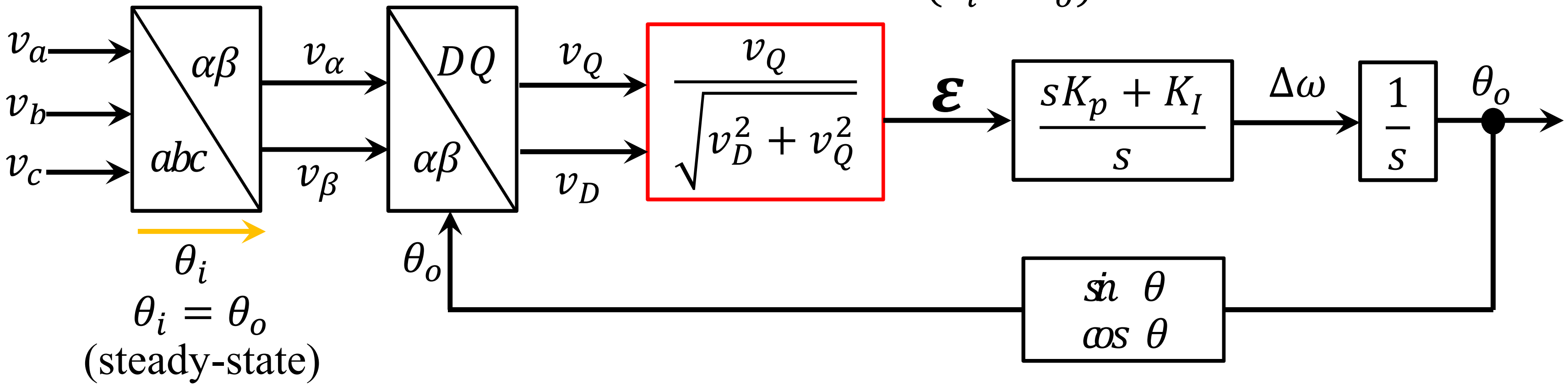
$$\cos(\omega_i + \theta_i) \cdot \cos(\omega_i + \theta_o)$$

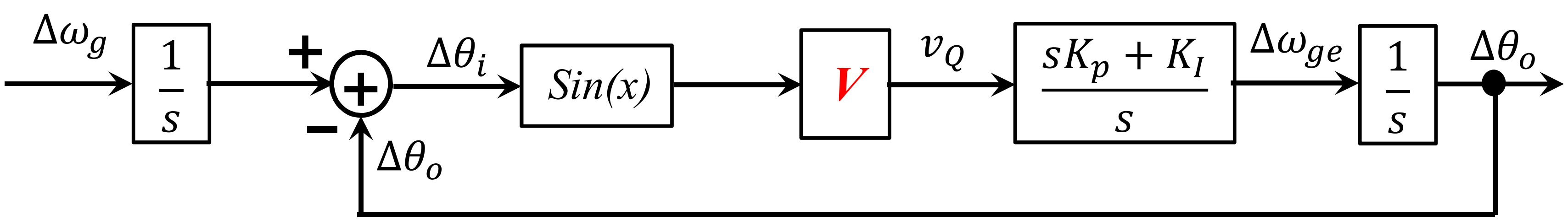
$$\frac{1}{2}\left[\cos\left((\omega_i - \omega_o)t + (\theta_i - \theta_o)\right) + \boldsymbol{\cos\left((\omega_i + \omega_o)t + (\theta_i + \theta_o)\right)}\right]$$

3 Phase Locked Loop (PLL) – ANS (Amplitude Normalization Scheme)

$\varepsilon = (\theta_i - \theta_o): \Delta\theta \ll 1$

$\varepsilon = \sin(\theta_i - \theta_o)$





Frequency ramp

$e_{steady}^{\Delta\omega_g} = \sin^{-1}\left(\frac{A}{VK_I}\right)$

$A = sJ\Delta\omega_g$

J = inertia

$$G_{PLL} = \frac{\Delta\omega_{ge}}{\Delta\omega_g} = \frac{VK_p s + VK_I}{s^2 + VK_p s + VK_I}$$

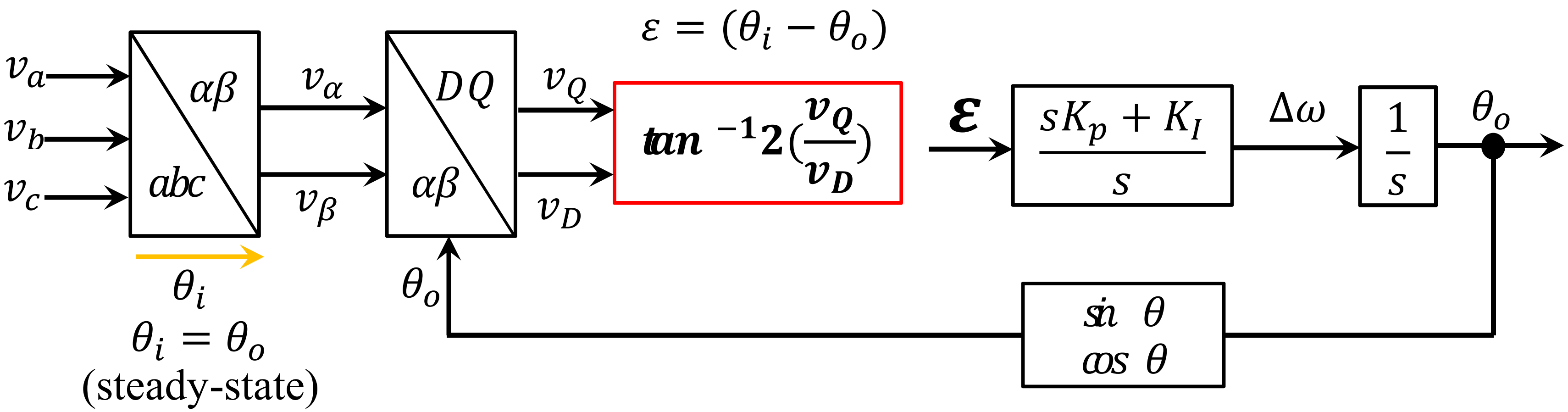
$$G_{PLL} = \frac{\Delta\theta_o}{\Delta\theta_i} = \frac{VK_p s + VK_I}{s^2 + VK_p s + VK_I}$$

V is gain in dynamic behavior

ANS is decoupling function

$\tan\left(\frac{v_\beta}{v_\alpha}\right)^{-1}$ $\tan\left(\frac{v_\beta}{V}\right)^{-1}$

3 Phase Locked Loop (PLL) – ANS (Amplitude Normalization Scheme)



$$\tan^{-1}\left(\frac{v_Q}{v_D}\right) = (\theta_i - \theta_o)$$

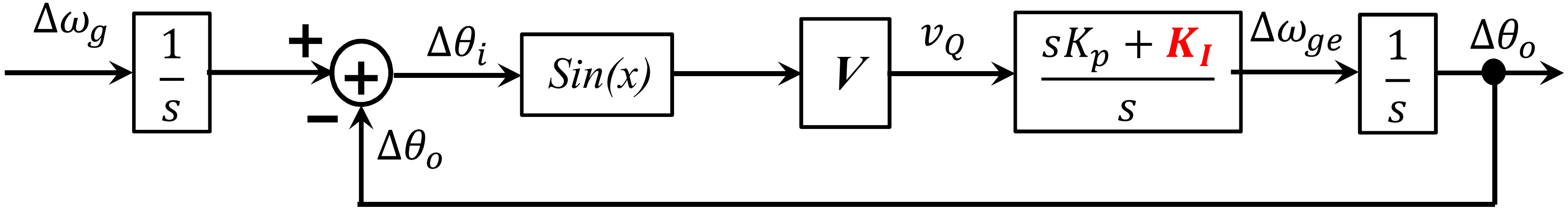
$$\frac{v_Q}{\sqrt{v_D^2 + v_Q^2}} = \sin\ (\theta_i - \theta_o)$$

$$\frac{v_Q}{v_D} = \tan\ (\theta_i - \theta_o)$$

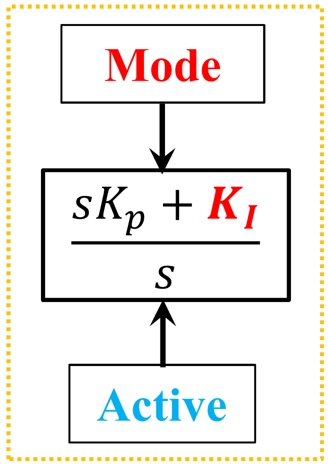
$$v_Q = \sin\ (\theta_i - \theta_o)$$

$$atan2\ \left(\frac{v_Q}{v_D}\right) = \begin{cases} \tan^{-1}\left(\frac{v_Q}{v_D}\right) : v_D > 0 \\ \tan^{-1}\left(\frac{v_Q}{v_D}\right) : v_D > 0 \text{ and } v_Q \geq 0 \\ \tan^{-1}\left(\frac{v_Q}{v_D}\right) : v_D > 0 \text{ and } v_Q < 0 \\ +\frac{\pi}{2} : v_D = 0 \text{ and } v_Q > 0 \\ -\frac{\pi}{2} : v_D = 0 \text{ and } v_Q < 0 \\ 0 : v_D = 0 \text{ and } v_Q = 0 \end{cases}$$

3 Phase Locked Loop (PLL) – ANS (Amplitude Normalization Scheme)



$$H_0(s) = \frac{G_{PLL}}{s + G_{PLL}} = \frac{K_D K_0 \left[\frac{\tau_2 s + 1}{\tau_1 s}\right]}{s + K_D K_0 \left[\frac{\tau_2 s + 1}{\tau_1 s}\right]} = \frac{K_D K_o[\tau_2 s + 1]}{\tau_1 s^2 + K_D K_o[\tau_2 s + 1]}$$



$$H_0(s) = \frac{K_D K_o[\tau_2 s + 1]}{\tau_1 s^2 + K_D K_o[\tau_2 s + 1]} = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

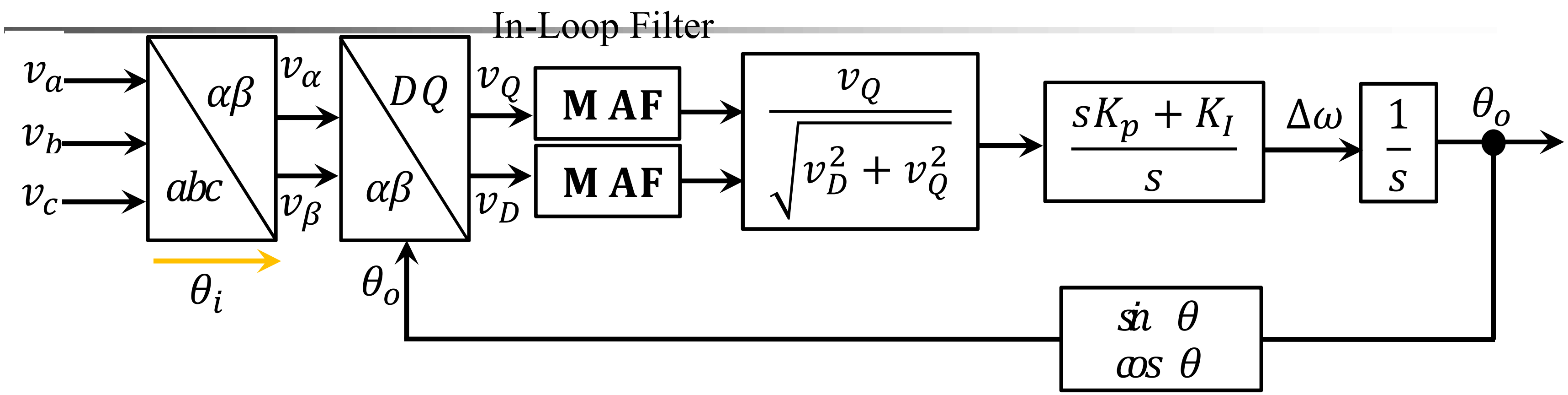
$$H_e(s) = \frac{s^2 K_D}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n = \sqrt{\frac{K_D K_o}{\tau_1}}$$

$$\zeta = \frac{\tau_2 \omega_n}{2}$$

$$\begin{cases} K_o = 0 : \; H_o(s) = 0 \\ \tau_1 = 0 : \omega_n = \infty \\ \tau_1 = \infty : \omega_n = 0 \\ \boldsymbol{\tau_2 = 0 : \zeta = 0} \\ \boldsymbol{\tau_2 = \infty : \zeta = \infty} \end{cases}$$

3-Phase Locked Loop (PLL) - Harmonics



MAF (Moving Average Filter): Linear Phase Filter,

Pass the DC component and blocks Frequencies of integer multiples of $(1/T_\omega)$

$$G_{MAF}(s) = \frac{1 - e^{-T_\omega s}}{T_\omega s}$$

Filtering Capability → Up   Slow down Dynamics → Large phase delay

$T = T_\omega$: T is PLL sampling time and grid harmonic is unknown → DC and All harmonics

$T_\omega = T/6$ or T/2 : odd-order harmonics and non-triple odd harmonic.

MAF+PI controller

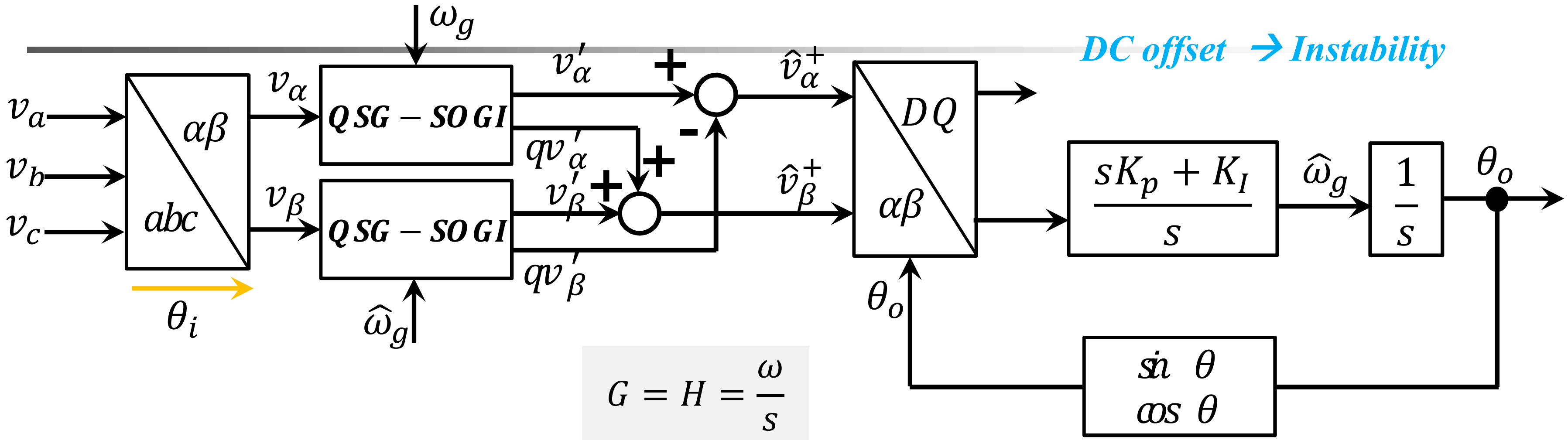
$$\left[\frac{1}{\tau s + 1} \cdot \frac{sK_P + K_I}{s}\right]$$

PID controller

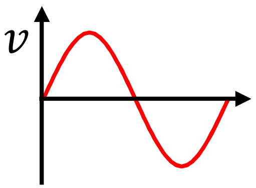
$$\frac{s^2 K_D + sK_I + K_P}{s}$$

$$\begin{cases} MAF + PI\ controller \\ PID\ control \\ Out-Loop\ + PLL \\ Quasi\ - P\ controller \end{cases}$$

3-Phase Locked Loop (PLL) - DSOGI

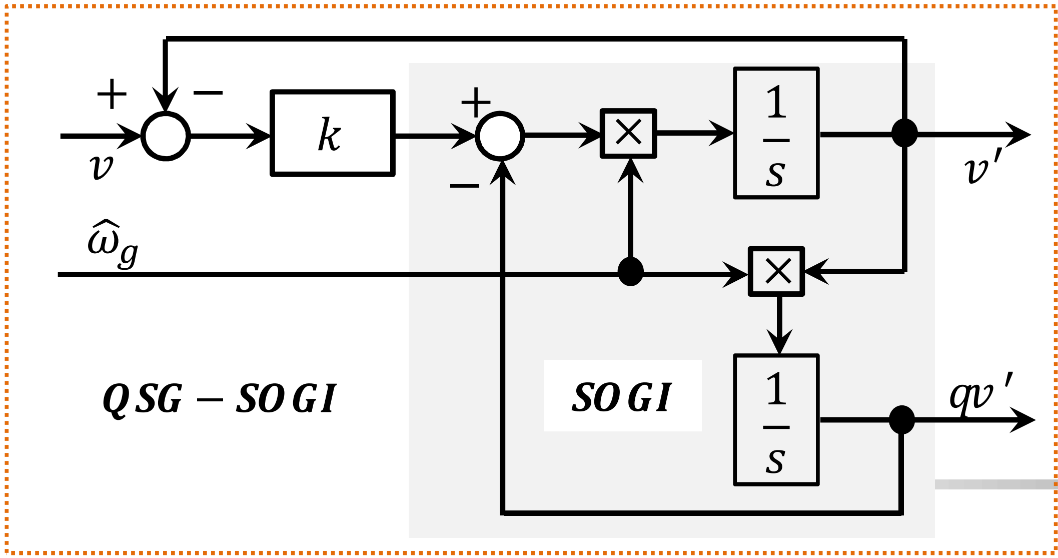


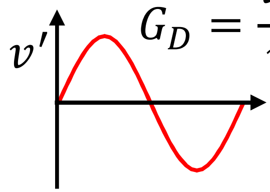
$$G = H = \frac{\omega}{s}$$



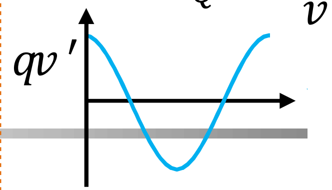
$$SOGI = \frac{G}{1 + GH} = \frac{\frac{\omega}{s}}{1 + \frac{\omega}{s} \cdot \frac{\omega}{s}} = \frac{\omega \cdot s}{s^2 + \omega^2} = \boldsymbol{\omega \cdot \cos(\omega t)}$$

By B. Berger, EPE, 2001





$$G_D = \frac{v'}{v} = \frac{k\omega s}{s^2 + k\omega s + \omega^2}$$

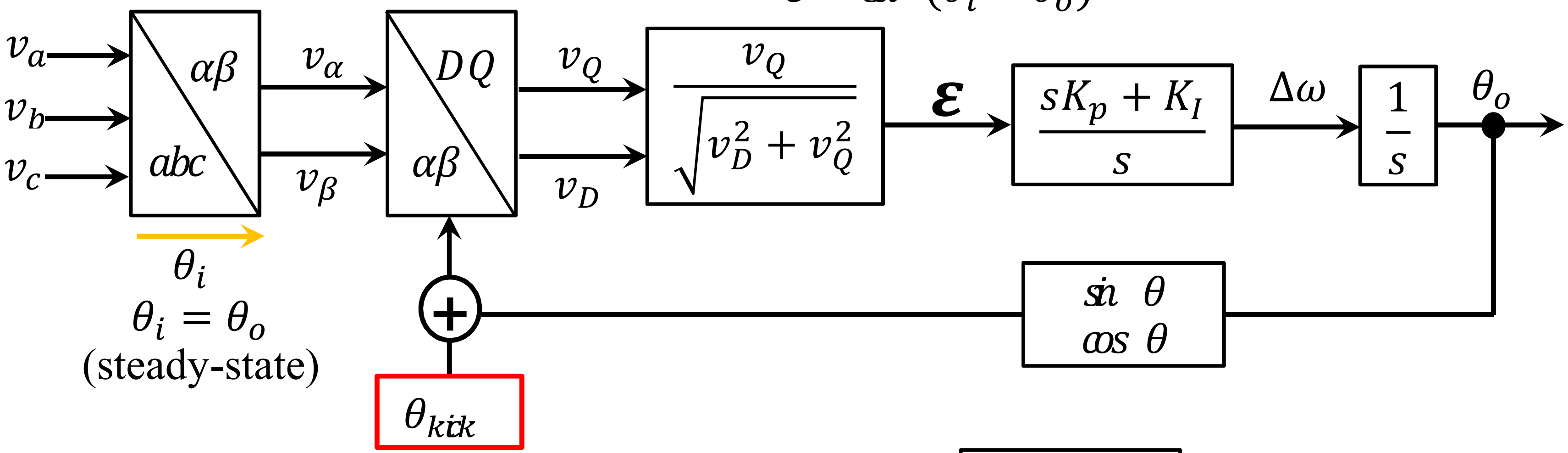


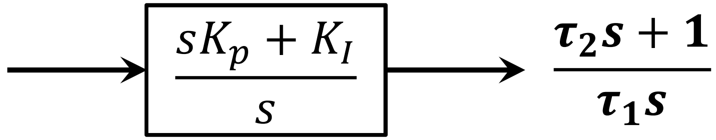
$$G_Q = \frac{qv'}{v} = \frac{k\omega^2}{s^2 + k\omega s + \omega^2}$$

3-Phase Locked Loop (PLL)

$\varepsilon = (\theta_i - \theta_o) : \Delta\theta \ll 1$

$\varepsilon = \sin(\theta_i - \theta_o)$

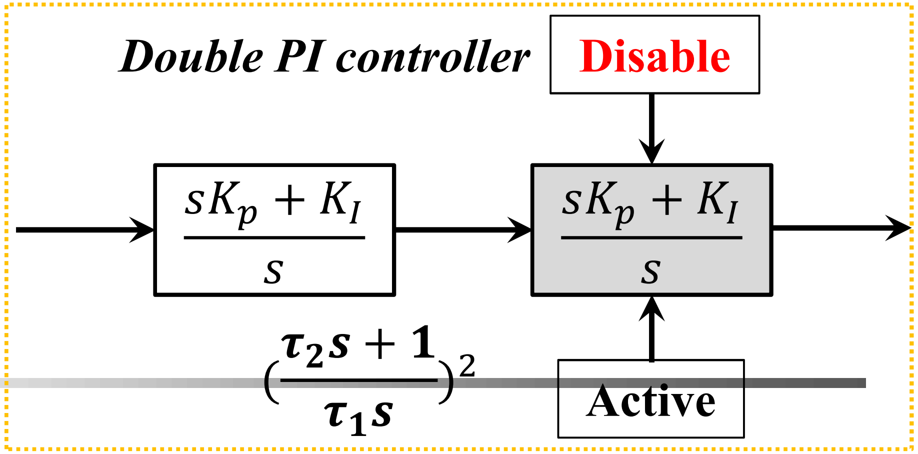




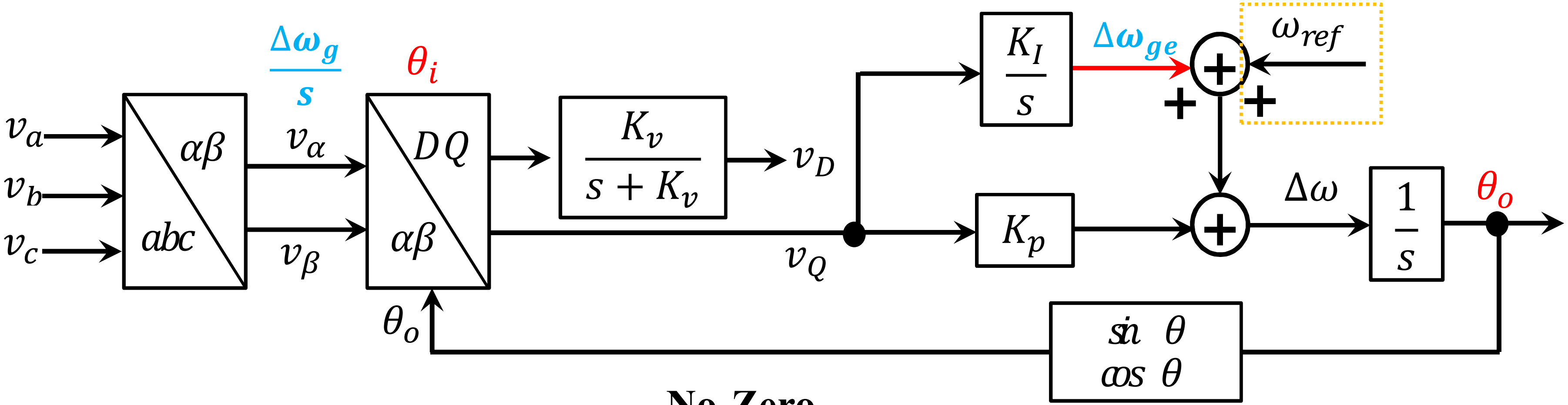
$$\frac{K}{s^3}, \frac{K}{s^2(\tau_1 s + 1)}, \frac{K(\tau_1 s + 1)}{s^3} : unstable$$

$$\left(\frac{\tau_2 s + 1}{\tau_1 s}\right)^2 : stable$$

Tracking frequency ramp with Zero error
Negative gain margin – ANS : ***vital***
Low loop gain(voltage low) : ***instability***
(voltage low : voltage sag)

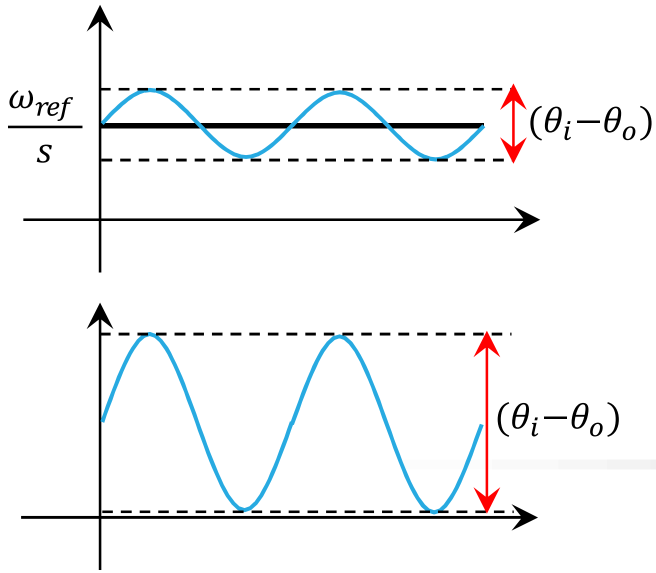


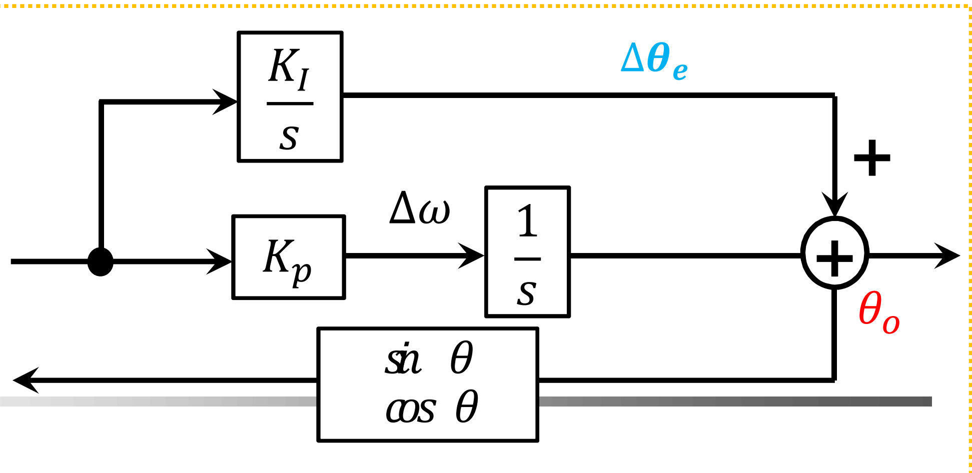
3 Phase Locked Loop (PLL) – (Modified SRF-PLL : Quasi PI controller)



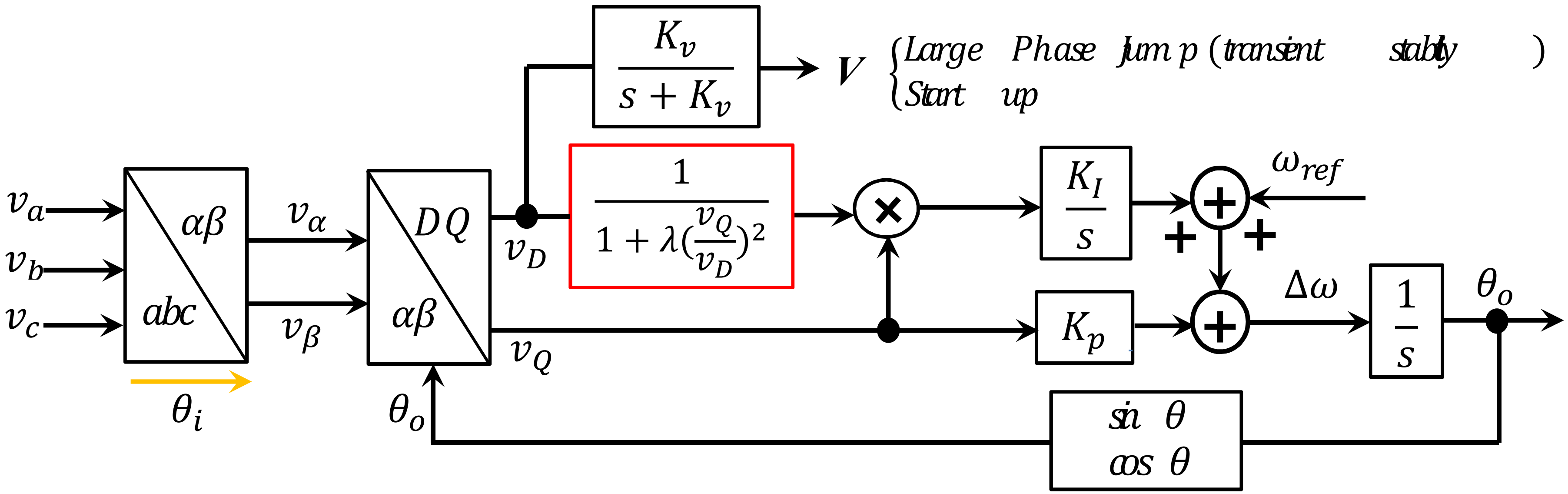
No-Zero

$$G_{PLL} = \frac{\Delta\omega_{ge}}{\Delta\omega_g} = \frac{\boldsymbol{V}K_p s + \boldsymbol{V}K_I}{s^2 + \boldsymbol{V}K_p s + \boldsymbol{V}K_I} \qquad G_{PLL} = \frac{\Delta\omega_{ge}}{\Delta\omega_g} = \frac{\boldsymbol{V}K_I}{s^2 + \boldsymbol{V}K_p s + \boldsymbol{V}K_I} \qquad G_{PLL} = \frac{\Delta\theta_o}{\Delta\theta_i} = \frac{\boldsymbol{V}K_p s + \boldsymbol{V}K_I}{s^2 + \boldsymbol{V}K_p s + \boldsymbol{V}K_I}$$



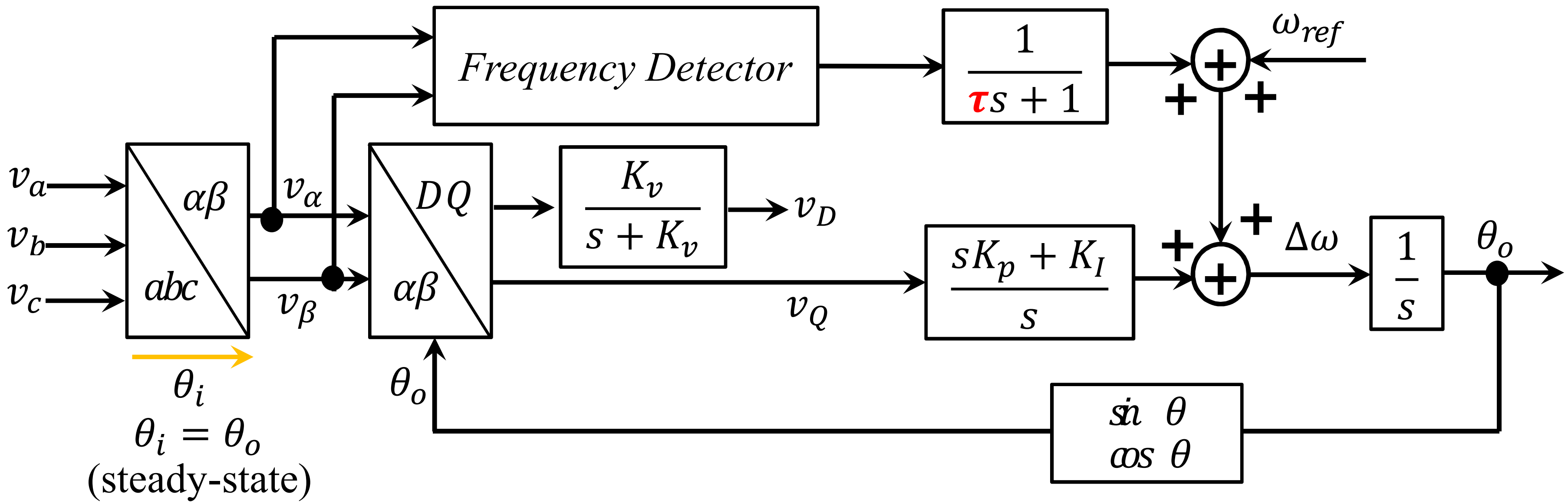


3-Phase Locked Loop (PLL) – Adaptive frequency estimation loop



IEEE Golestan!

3 Phase Locked Loop (PLL) – (Modified SRF-PLL-Quasi-Double PI controller)



$(\frac{\tau_2 s + 1}{\tau_1 s})^2$: *stable*

τ= 3 x TST

(Transient Stability Time)

$3\tau = 0.05 = 5\%$

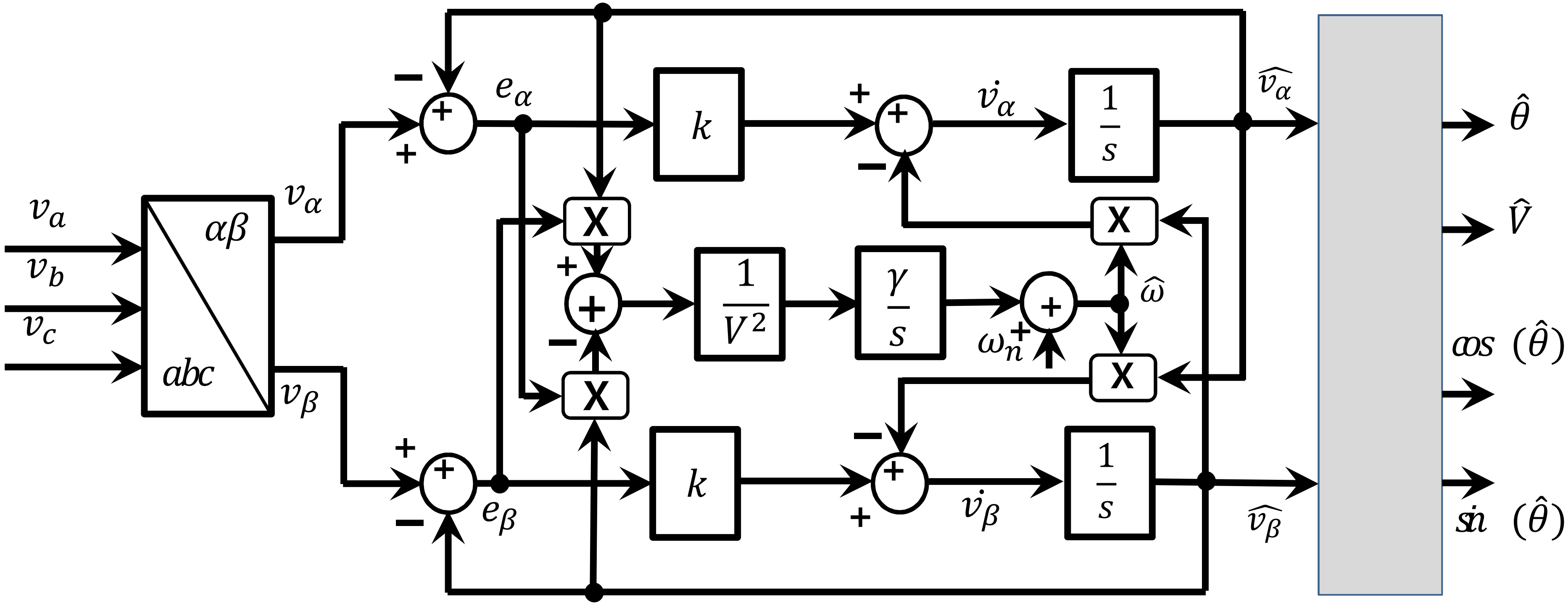
Feedforward Loop

$\tan^{-1}\left(\frac{v_\beta}{v_\alpha}\right) \cdot \frac{d}{dt}$ = ***Frequency Detector***

Tracking frequency ramp with Zero error
Negative gain margin – ANS : ***vital***
Low loop gain(voltage low) : ***instability***

Tracking frequency ramp with Zero error
Negative gain margin – ANS : ***recommended***
Low loop gain(voltage low) : ***stability***

3 - Frequency Locked Loop (FLL)

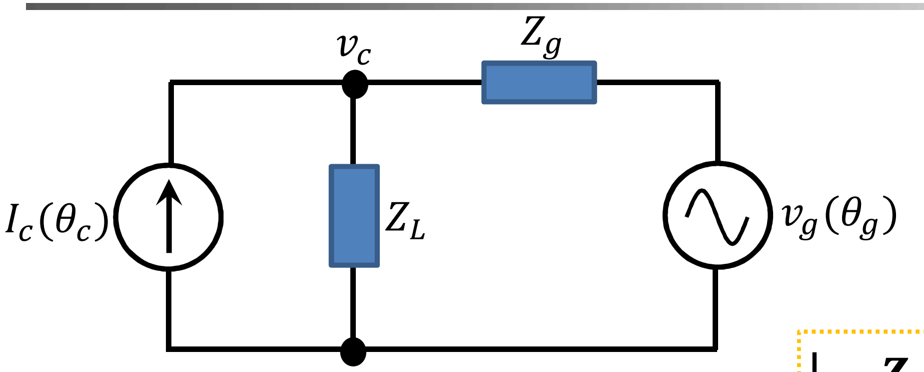


$$\hat{\theta} = \operatorname{atan}\left(\frac{\widehat{v_\alpha}}{\widehat{v_\beta}}\right) \qquad \hat{V} = \sqrt{\widehat{v_\alpha}^2 + \widehat{v_\beta}^2}$$

$$\sin(\hat{\theta}) = \frac{\widehat{v_\beta}}{\hat{V}}$$

$$\cos(\hat{\theta}) = \frac{\widehat{v_\alpha}}{\hat{V}}$$

3-Phase Locked Loop (PLL) – Self and Grid Synchronization Loop



$$v_c = v_{c1} + v_{c2}$$

$$v_c = \frac{Z_L}{Z_L + Z_g} v_g + \frac{Z_L Z_g}{Z_L + Z_g} I_c$$

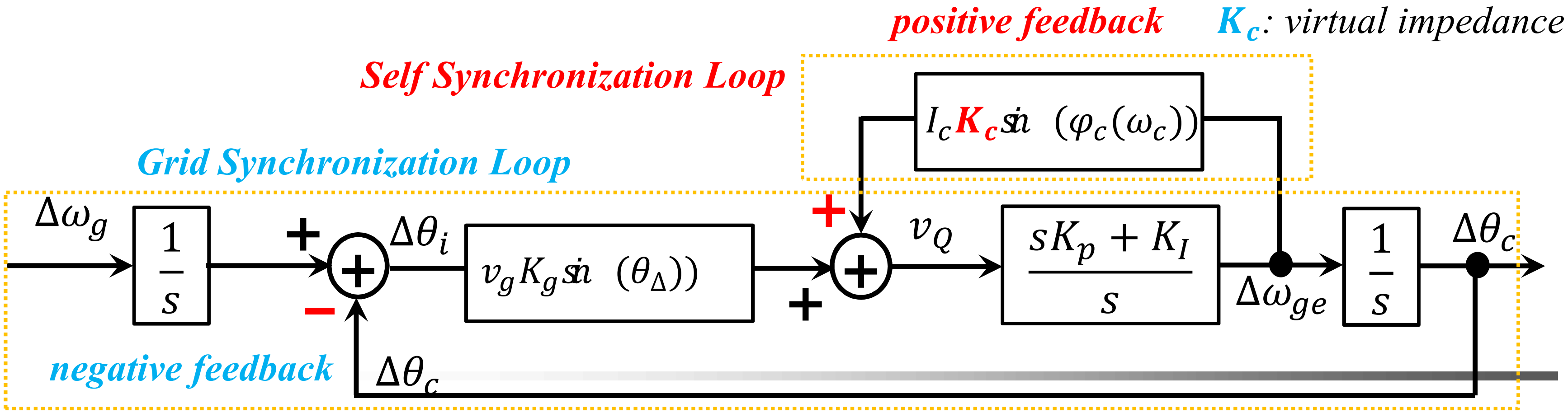
$$\varphi_g = \angle[\frac{Z_L}{Z_L + Z_g}] \qquad \varphi_c = \angle[\frac{Z_L Z_g}{Z_L + Z_g}]$$

$$\boldsymbol{v_c} = \left|\frac{\boldsymbol{Z_L}}{\boldsymbol{Z_L + Z_g}}\right| \cdot \boldsymbol{v_g}\, \boldsymbol{e^{j(\theta_g+\varphi_g)}} + \left|\frac{\boldsymbol{Z_L Z_g}}{\boldsymbol{Z_L + Z_g}}\right| \cdot \boldsymbol{I_c e^{j(\theta_c+\varphi_c)}}$$

Grid Synchronization Loop ***Self Synchronization Loop***

$$\varphi_g(\omega_g) = \angle[\frac{Z_L(\omega_g)}{Z_L(\omega_g) + Z_g(\omega_g)}]$$

$$\varphi_c(\omega_c) = \angle[\frac{Z_L(\omega_c) Z_g(\omega_c)}{Z_L(\omega_c) + Z_g(\omega_c)}] \qquad K_g(\omega_g) = \left|\frac{Z_L(\omega_g)}{Z_L(\omega_g) + Z_g(\omega_g)}\right| \qquad K_c(\omega_c) = \left|\frac{Z_L(\omega_c) Z_g(\omega_c)}{Z_L(\omega_c) + Z_g(\omega_c)}\right|$$

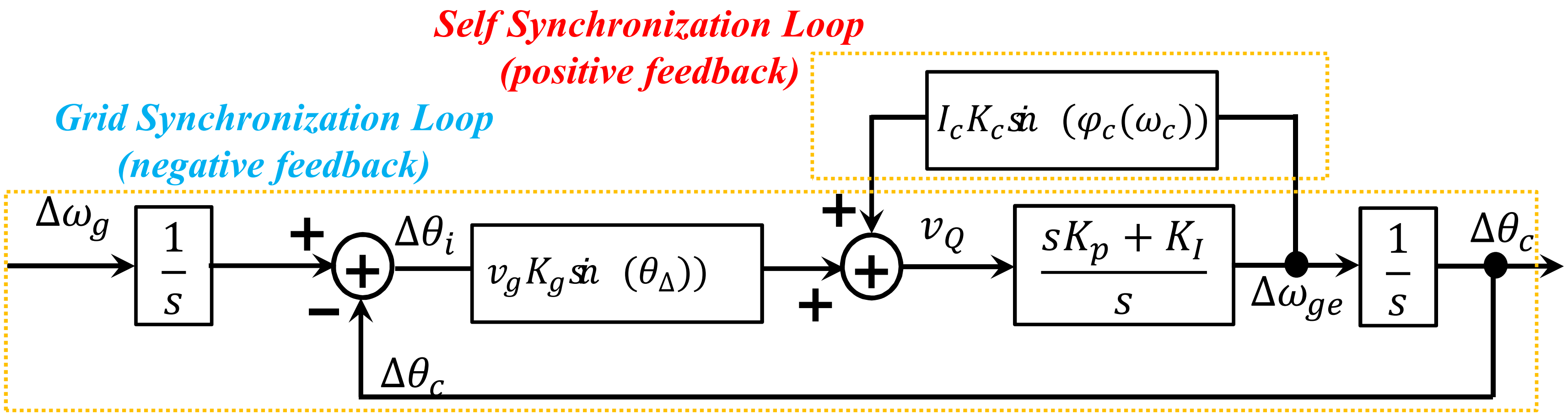


3-Phase Locked Loop (PLL) – Self and Grid Synchronization Loop

$$\boldsymbol{v_c} = \left|\frac{\boldsymbol{Z_L}}{\boldsymbol{Z_L} + \boldsymbol{Z_g}}\right| \cdot \boldsymbol{v_g}\, \boldsymbol{e^{j(\theta_g+\varphi_g)}} + \left|\frac{\boldsymbol{Z_L Z_g}}{\boldsymbol{Z_L} + \boldsymbol{Z_g}}\right| \cdot \boldsymbol{I_c e^{j(\theta_c+\varphi_c)}}$$

Grid Synchronization Loop ***Self Synchronization Loop***

$$\varphi_c(\omega_c) = \angle\left[\frac{Z_L(\omega_c)Z_g(\omega_c)}{Z_L(\omega_c) + Z_g(\omega_c)}\right] \quad K_g(\omega_g) = \left|\frac{Z_L(\omega_g)}{Z_L(\omega_g) + Z_g(\omega_g)}\right| \quad K_c(\omega_c) = \left|\frac{Z_L(\omega_c)Z_g(\omega_c)}{Z_L(\omega_c) + Z_g(\omega_c)}\right|$$



$$v_c = K_g \cdot v_g \cdot e^{j(\theta_g+\varphi_g)} + K_c \cdot I_c \cdot e^{j(\theta_c+\varphi_c)}$$

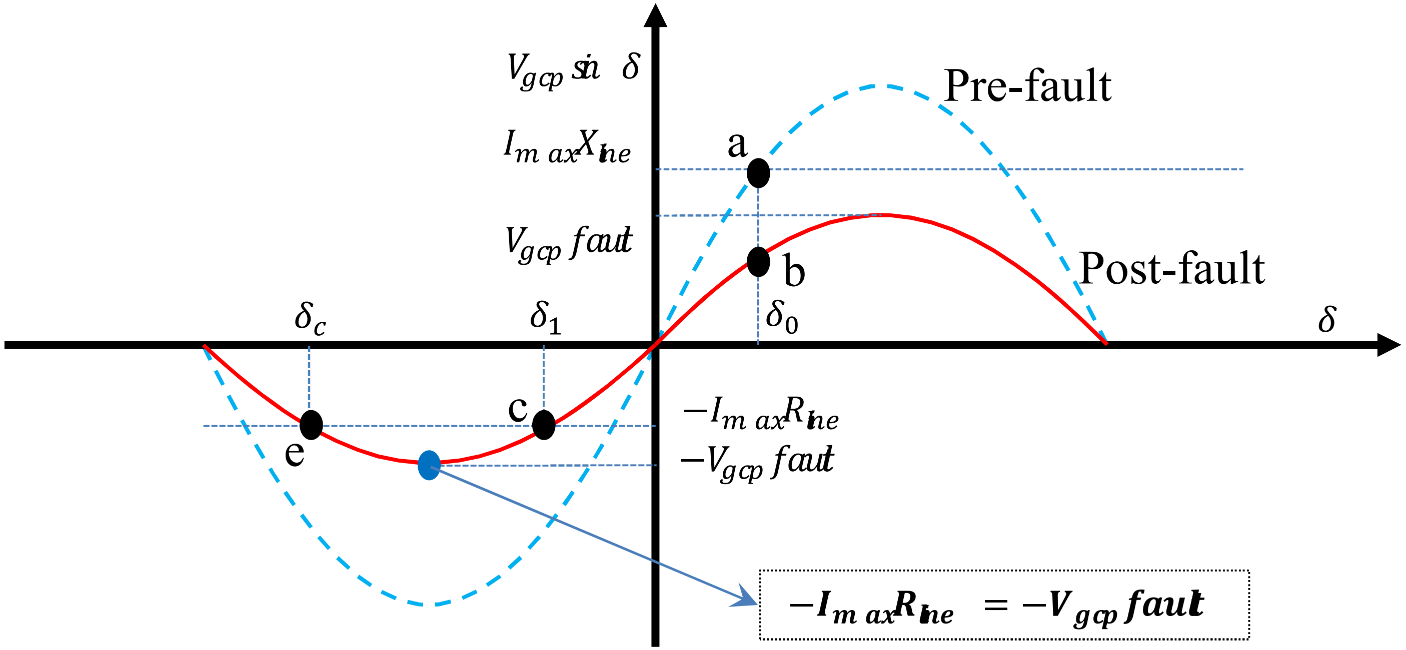
$$\boldsymbol{e^{j\theta_c}} \otimes \begin{cases} K_g \cdot v_g \cdot e^{j(\theta_g+\varphi_g-\boldsymbol{\theta_c})} \\ K_c \cdot I_c \cdot e^{j(\theta_c+\varphi_c+\boldsymbol{\theta_c})} \end{cases}$$

$$v_{iq} = K_g v_g \sin(\theta_g + \varphi_g - \theta_c) + K_c I_c \sin(\varphi_c)$$

$$\sin\left(\varphi_g - \boldsymbol{\theta_c^{ref}}\right) = -\frac{K_c I_c \sin(\boldsymbol{\varphi_c})}{K_g v_g} = \begin{cases} \textbf{Stability} \\ Z_{const} > P_{const} \\ 80\% : 70\% \\ f_{pI} > f_{pI}\ (20Hz > 2Hz) \\ 72\% : 70\% \end{cases}$$

$x: \varphi_c$ and $y: \theta_c^{ref}$

2. Transient Instability



Pre-fault Condition

$$I_dX_{line} + I_qR_{line} = V_{gcp}\sin\delta$$

$$\underbrace{I_dX_{line}}_{Active\ Power} + \underbrace{I_qR_{line}}_{Reactive\ Power} \le V_{gcp}$$

$$\underbrace{I_dX_{line}}_{I_d=max} + \underbrace{I_qR_{line}}_{I_q=0} = I_{max}X_{line} = V_{gcp}\sin\delta_0$$

Steady-State

Post-fault Condition

$$\underbrace{I_dX_{line}}_{I_d=0} + \underbrace{I_qR_{line}}_{I_q=-I_{max}} = -I_{max}R_{line}$$

$$-I_{max}R_{line} < -(V_{gcp\,fault}\sin\delta)_{max} = -V_{gcp\,fault}$$

Instability Condition

II. State Condition Control

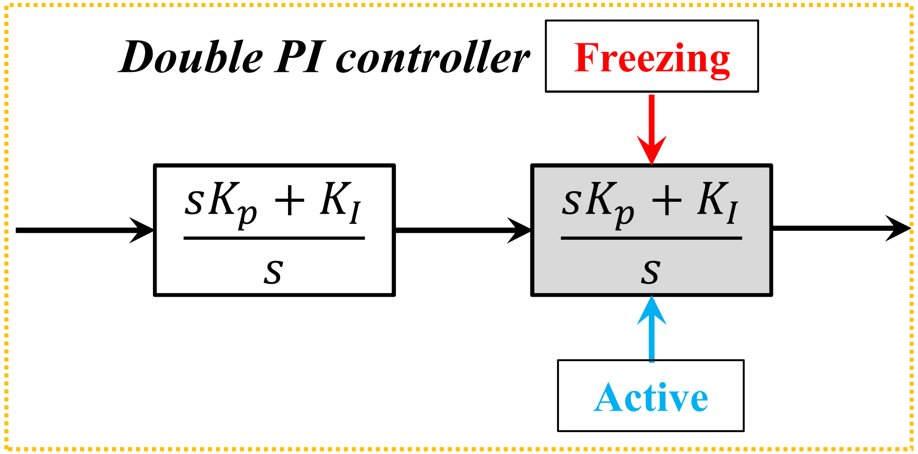
State Condition Control

$$\textbf{Controler} = \begin{cases} \textbf{Freezing} : \text{Present value} \\ \textbf{Disable} : \begin{cases} \text{Output} : 0\ (\text{Normal}) \\ K_I : 0\ (K_P : \text{Design value}) \\ K_I : \text{some values}\ (K_P : \text{Design value}) \end{cases} \\ \textbf{Gain Mode} \begin{cases} K_I = 0 \\ K_P : 1 \end{cases} \end{cases}$$

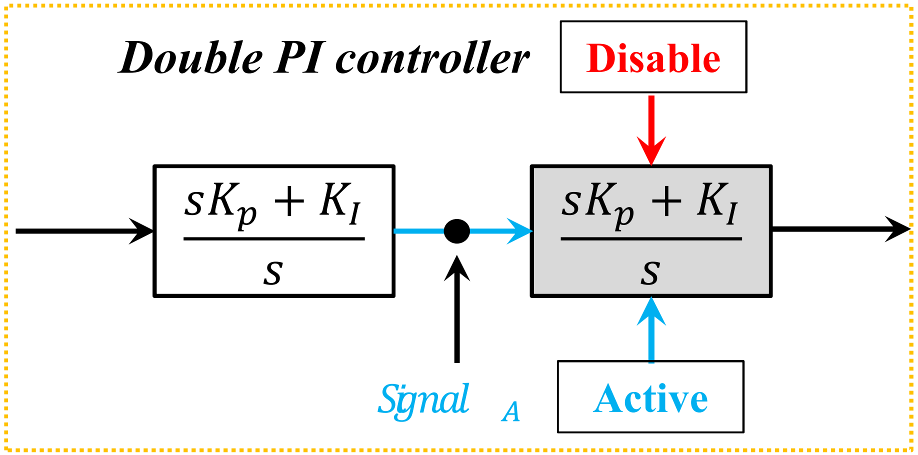
$$\textbf{Limiter} = \begin{cases} \textbf{Freezing} : \text{Present value} \\ \textbf{Disable} : \begin{cases} 0\ (\text{Normal}) \\ +1\ (\text{Max value}) \\ -1\ (\text{Min value}) \end{cases} \\ \textbf{Upper Limiter} : \begin{cases} \textbf{MAXselector} \\ P_{U1} \\ P_0 \end{cases} \\ \textbf{Lower Limiter} : \begin{cases} \textbf{MIN}_{\textbf{selector}} \\ P_{L1} \\ P_0 \end{cases} \end{cases}$$

$$\textbf{Reference} = \begin{cases} \text{Normal State} : +1 \\ \text{Emergency} : 0 \\ \text{Power Reversal} : -1 \\ \text{LVRT or FRT} : \\ \rightarrow \text{Determined value} \end{cases}$$

Controller-2 - State Condition Control



$Output = present\ value$

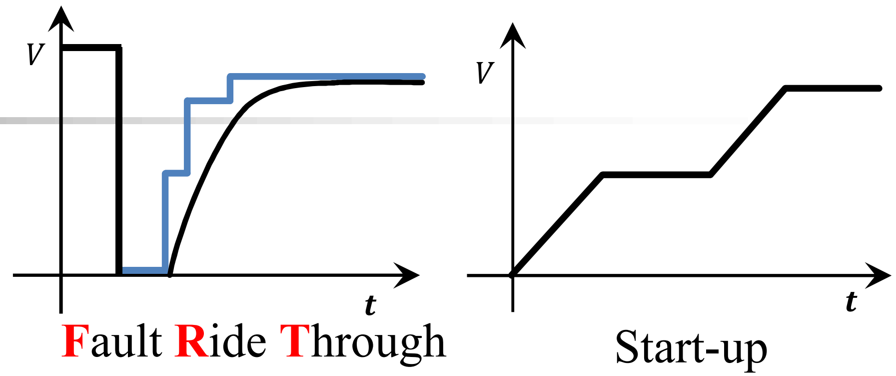


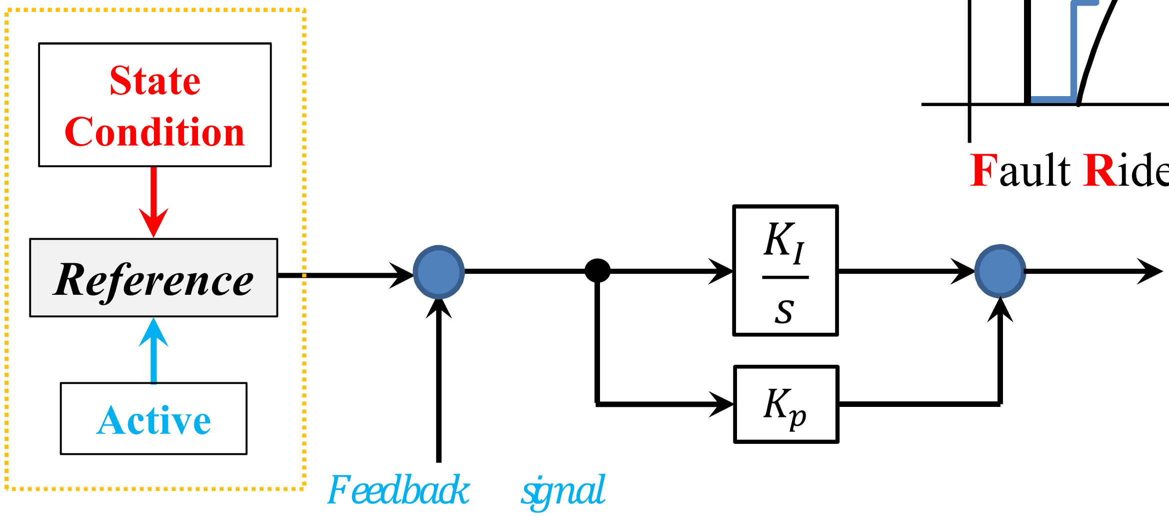
$Output = 0$

Or

$Output = Signal_A$

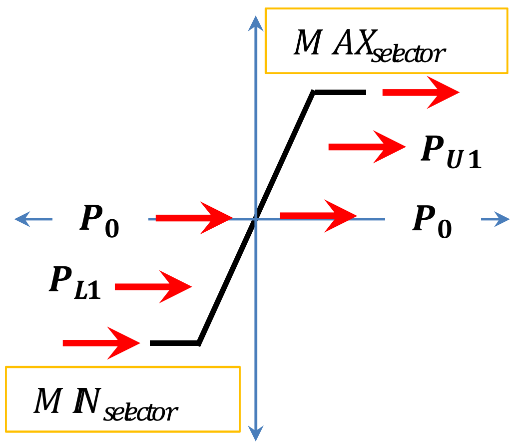
Reference – State Condition Control





Limiter (2) – State Condition Control

- To protect HVDC system
  - Overvoltage, Thermal, Current
- To coordinate the stability of the system
  - Frequency, Generator, Inertia
  - Voltage, Reactive Power
- To coordinate the Trip and Block signals
  - Test, Trip, Block



$$\boldsymbol{Limiter} = \begin{cases} Freezing : \begin{cases} Temporary\ faults \\ Sensor\ error \end{cases} \\ Disable : \begin{cases} Trip\ signal \\ Test\ signal \\ Block\ signal \end{cases} \\ Active\ 1\ for\ Freezing\ signal \\ Active\ 2\ for\ Disable\ signal \end{cases}$$

$$\boldsymbol{P_0} = \begin{cases} Test\ and\ Block\ signal \\ Coordinated\ with\ Trip \\ Time : Immediately \\ Limiter\ Output : 0 \end{cases}$$

$$\boldsymbol{P_{U1} and\ P_{L1}} = \begin{cases} Control\ range \\ Coordinated\ for\ stably \\ Time : Designed\ value \\ Limiter\ output : \\ \rightarrow some\ value \end{cases}$$

II. PLL and AC network

3-Phase Locked Loop (PLL) – Time Delay effect (1)

$$v_\alpha = v_D \cos\left(\omega_0 t + \theta_o + \theta_{Delay}\right) - \boxed{v_Q \sin\left(\omega_o t + \theta_o\right)} = \mathbf{0} \qquad \begin{cases} v_D = 1 \\ v_Q = 0 \end{cases}$$

$$v_\alpha = \cos\left(\omega_0 t + \theta_o + \boldsymbol{\theta_{Delay}}\right) = \cos\left(\omega_0 t + \theta_0\right) \cdot \underbrace{\cos\left(\boldsymbol{\theta_{Delay}}\right)}_{\mathbf{A}} - \sin\left(\omega_o t + \theta_o\right) \cdot \underbrace{\sin\left(\boldsymbol{\theta_{Delay}}\right)}_{\mathbf{B}}$$

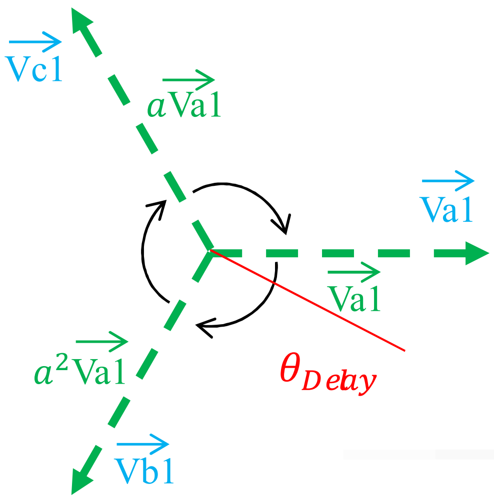
$$v_\alpha = \boldsymbol{A} \cdot \cos\left(\omega_0 t + \theta_0\right) - \boldsymbol{B} \cdot \sin\left(\omega_o t + \theta_o\right)$$

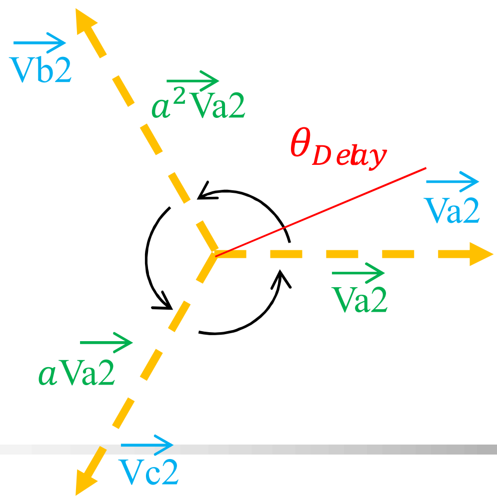
$$v_\alpha = \frac{A + jB}{2} e^{j\omega t} + \frac{A - jB}{2} \boldsymbol{e^{-j\omega t}}$$

Negative sequence

$$1[ms] \begin{cases} 60Hz: \Delta\theta : 20.4^0 \\ 50Hz: \Delta\theta : 18^0 \end{cases}$$

$$\mathbf{32\,[us]} \begin{cases} 60Hz: \Delta\theta : 0.8^0 \\ 50Hz: \Delta\theta : 0.6^0 \end{cases}$$





[$\boldsymbol{v_Q} = 0$] control

sideband harmonics

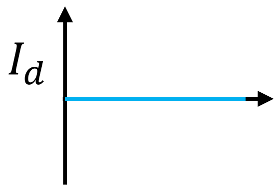
3-Phase Locked Loop (PLL) - D/Q control effect - $v_q = 0$

$$v_\alpha = \mathbf{A}\cos(\omega_0 t + \theta_0) - \mathbf{B}\sin(\omega_o t + \theta_o)$$

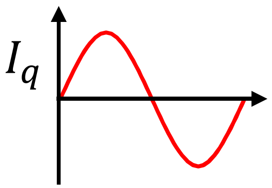
$$v_\alpha = \frac{A + jB}{2} e^{j\omega t} + \frac{A - jB}{2} e^{-j\omega t}$$

Sideband effect

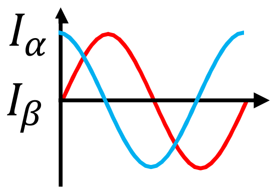
$$I_d + jI_q = A_0 + A_1 e^{j\omega t} + \boldsymbol{A_2 e^{-j\omega t}}$$

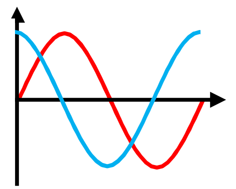


$\boldsymbol{\omega_0 t}$

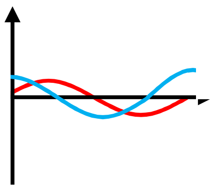


$\boldsymbol{e^{j(\omega_1 + \omega_D)t}}$

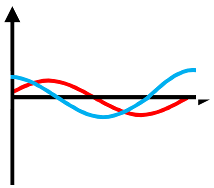




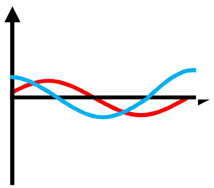
$\boldsymbol{\omega_1 t}$



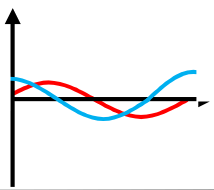
$\boldsymbol{(\omega_1 + \omega_0 + \omega_D)t}$



$\boldsymbol{(\omega_1 - \omega_0 + \omega_D)t}$



$\boldsymbol{(\omega_1 + \omega_0 - \omega_D)t}$



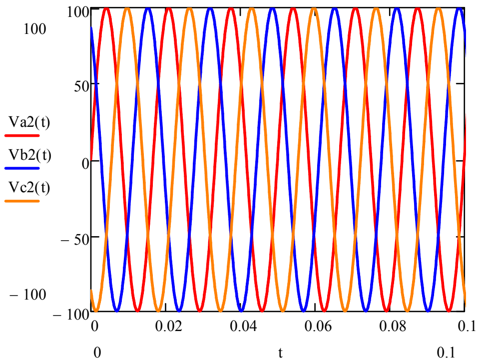
$\boldsymbol{(\omega_1 + \omega_0 - \omega_D)t}$

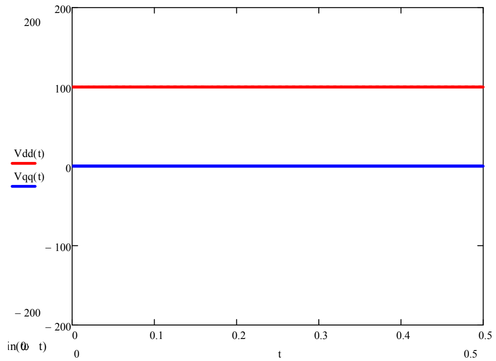
3-Phase Locked Loop (PLL) – Time Delay effect (2)

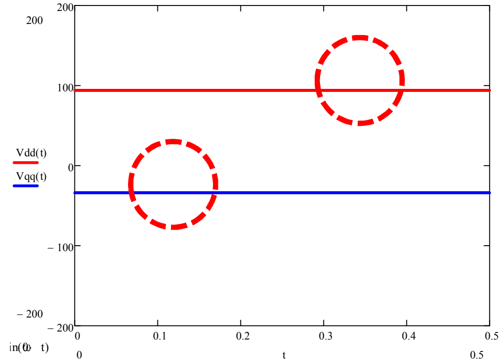
$$v_\alpha = v_D \cos\left(\omega_0 t + \theta_o + \boldsymbol{\theta_{Delay}}\right) - \boldsymbol{v_Q} \sin\left(\omega_o t + \theta_o\right)$$

$$v_\beta = v_D \sin\left(\omega_0 t + \theta_o + \boldsymbol{\theta_{Delay}}\right) + \boldsymbol{v_Q} \cos\left(\omega_o t + \theta_o\right)$$

$$\begin{cases} \theta_{Delay} : 20^0 : 1[ms] \\ v_Q : 0 \rightarrow -34.185 \\ v_D : 100 \rightarrow 93.975 \end{cases}$$



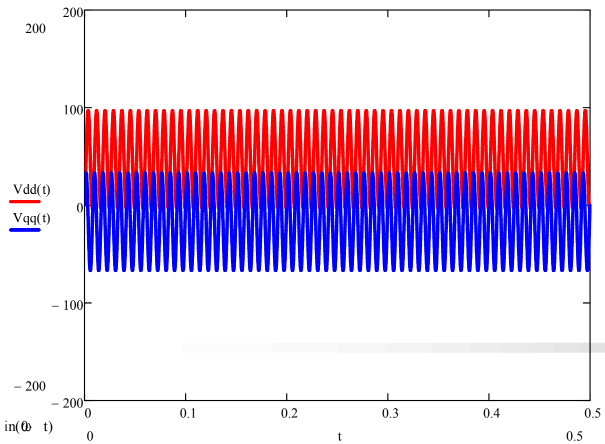


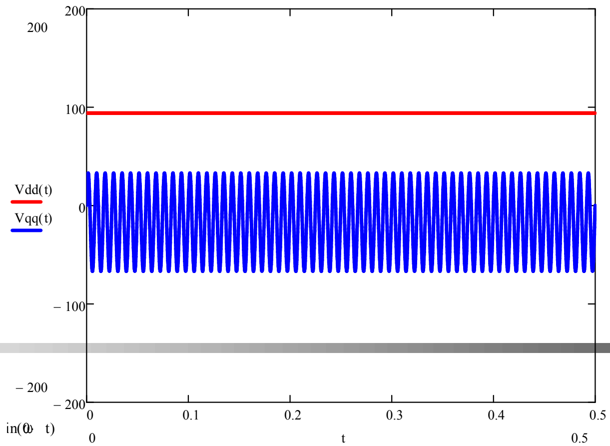


$$v_\alpha = v_D \cos\left(\omega_0 t + \theta_o + \boldsymbol{\theta_{Delay}}\right) - \boldsymbol{v_Q} \sin\left(\omega_o t + \theta_o\right)$$

$$v_\beta = v_D \sin\left(\omega_0 t + \theta_o + \boldsymbol{\theta_{Delay}}\right) + \boldsymbol{v_Q} \cos\left(\omega_o t + \theta_o\right)$$

$$\{\boldsymbol{v_Q = 0 \ control}$$





120Hz

3-Phase Locked Loop (PLL) – unbalance condition

$$v_\alpha = V\cos(\theta_i) + V[\frac{\beta+\gamma}{6}\cos(\theta_i) - \frac{\beta-\gamma}{2\sqrt{3}}\sin(\theta_i)]$$

$$v_\beta = -V\sin(\theta_i) + V[\frac{\beta-\gamma}{2\sqrt{3}}\cos(\theta_i) - \frac{\beta+\gamma}{2}\sin(\theta_i)]$$

$$v_a = V\cos(\theta)$$

$$v_b = V(1+\beta)\cos(\theta - \frac{2}{3}\pi)$$

$$v_c = V(1+\gamma)\cos(\theta + \frac{2}{3}\pi)$$

$$\boldsymbol{v_Q = v_\alpha \cos(\theta_o) + v_\beta \sin(\theta_o)}$$

$$\boldsymbol{v_Q = V\sin(\theta_i - \theta_o) - [\frac{\beta-\gamma}{2\sqrt{3}}(\cos(\theta_i)\cos(\theta_o) - \sin(\theta_i)\sin(\theta_o)) + \frac{\beta+\gamma}{6}(\sin(\theta_i)\cos(\theta_o) - \cos(\theta_i)\sin(\theta_o))]}$$

$$v_Q = V\sin(\theta_i - \theta_o) - V[\frac{\beta-\gamma}{2\sqrt{3}}\cos(\theta_i+\theta_o) + \frac{\beta+\gamma}{6}\sin(\theta_i+\theta_o)]$$

$$v_Q = V(\theta_i - \theta_o) - V\sqrt{(\frac{\beta-\gamma}{2\sqrt{3}})^2 + (\frac{\beta+\gamma}{6})^2} \cdot \cos\left(\theta_i + \theta_o - \tan^{-1}(\frac{1}{\sqrt{3}}\left(\frac{\beta+r}{\beta-\gamma}\right))\right)$$

unbalance

$$\boldsymbol{v_Q = -V\sqrt{(\frac{\beta-\gamma}{2\sqrt{3}})^2 + (\frac{\beta+\gamma}{6})^2} \cdot \cos\left(2\theta - \tan^{-1}(\frac{1}{\sqrt{3}}\left(\frac{\beta+r}{\beta-\gamma}\right))\right)} \quad (\theta_i = \theta_o)$$

3-Phase Locked Loop (PLL) – Voltage offset

$$v_\alpha = V\cos(\theta_i) + v_{\alpha 0} \qquad v_\beta = V\cos(\theta_i) + v_{\beta 0}$$

$$v_Q = \underline{-V\sin(\theta_i - \theta_o)} + \underline{v_{\alpha 0}\sin(\theta_o) + v_{\beta 0}\cos(\theta_o)}$$

$$\boldsymbol{v_Q = \underline{V\delta} + \underline{V_0\cos(\theta + \varphi_0)}}$$

$$V_0 = \sqrt{v_{\alpha 0}^2 + v_{\beta o}^2} \qquad \varphi_0 = -\tan^{-1}\left(\frac{v_{\beta 0}}{v_{\alpha 0}}\right)$$

$$\delta \cong V_{Q0}\cos(\theta + \varphi_0) \qquad V_{Q0} = -\frac{V_0}{V}$$

$$v_a = V\cos(\theta) + V_{a0}$$
$$v_b = V\cos\left(\theta - \frac{2}{3}\pi\right) + V_{b0}$$
$$v_c = V\cos\left(\theta + \frac{2}{3}\pi\right) + V_{c0}$$

$$v_{\alpha 0} = \frac{2}{3}(V_{a0} + V_{b0} + V_{c0})$$
$$v_{\beta 0} = \frac{1}{\sqrt{3}}(V_{c0} - V_{b0})$$

$$\begin{cases} \text{Grid faults : CT saturation} \\ \text{CT or PT and A/D conversion} \\ \text{Geomagnetic phenomena} \\ \text{Half wave rectification} \end{cases}$$

$$\begin{cases} \text{Power frequency oscillation} \\ \text{DC injection to AC grid} \end{cases}$$

$$\begin{cases} \text{Pre filtering} \\ \text{Control method} \\ \text{Prevention} \end{cases}$$

3-Phase Locked Loop (PLL) - D/Q control effect - $v_q = 0$

$$-V\cos(\omega_i + \theta_i)\sin(\omega_0 + \theta_o) + V\sin(\omega_i + \theta_i)\cos(\omega_0 + \theta_o)$$

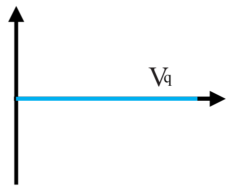
$$-\frac{1}{2}[\sin\left((\omega_i + \omega_o)t + (\theta_i + \theta_o)\right) - \sin((\omega_i - \omega_o)t + (\theta_i - \theta_o))]$$

$$\frac{1}{2}[\sin\left((\omega_i + \omega_o)t + (\theta_i + \theta_o)\right) + \sin((\omega_i - \omega_o)t + (\theta_i - \theta_o))]$$

$$\theta_i \neq \theta_o \quad \omega_i \neq \omega_o$$

$$\sin((\omega_i - \omega_o)t + (\theta_i - \theta_o))]$$

$$\rightarrow \sin(\omega_o t + \vartheta_o))]$$



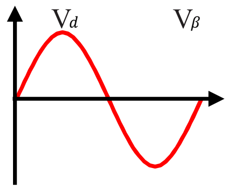
$$v_\alpha = v_D\cos(\omega_0 t + \theta_o) - \boldsymbol{v_Q}\sin(\omega_o t + \theta_o)$$

$$= V\sin((\omega_i + \boldsymbol{\Delta\omega})t + \theta_0)\cdot\cos(\omega_0 t + \theta_o) - \mathbf{0}\cdot\sin(\omega_o t + \theta_o)$$

$$= V[\sin((\omega_i)t + \theta_i) + \sin((\omega_i - \Delta\omega)t + \theta_i)]$$

$$\theta_i = \theta_o \quad \omega_i = \omega_o$$

$$v_Q = 0$$



$$v_\beta = v_D\sin(\omega_0 t + \theta_o) + \boldsymbol{v_Q}\cos(\omega_o t + \theta_o)$$

$$= V\sin(\omega_i t + \boldsymbol{\Delta\omega} + \theta_0)\cdot\sin(\omega_0 t + \theta_o) + \mathbf{0}\cdot\cos(\omega_0 t + \theta_o)$$

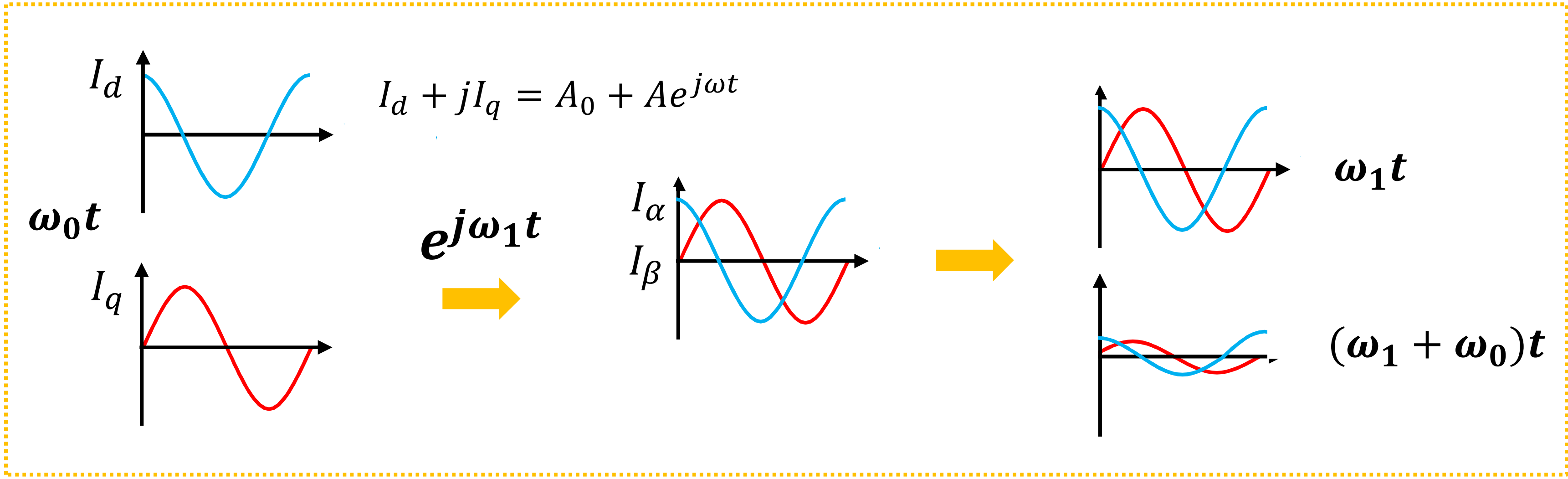
$$= V[\cos\left((\omega_i)t + (\theta_i)\right) - \cos((\omega_i - \Delta\omega)t + (\theta_i))]$$

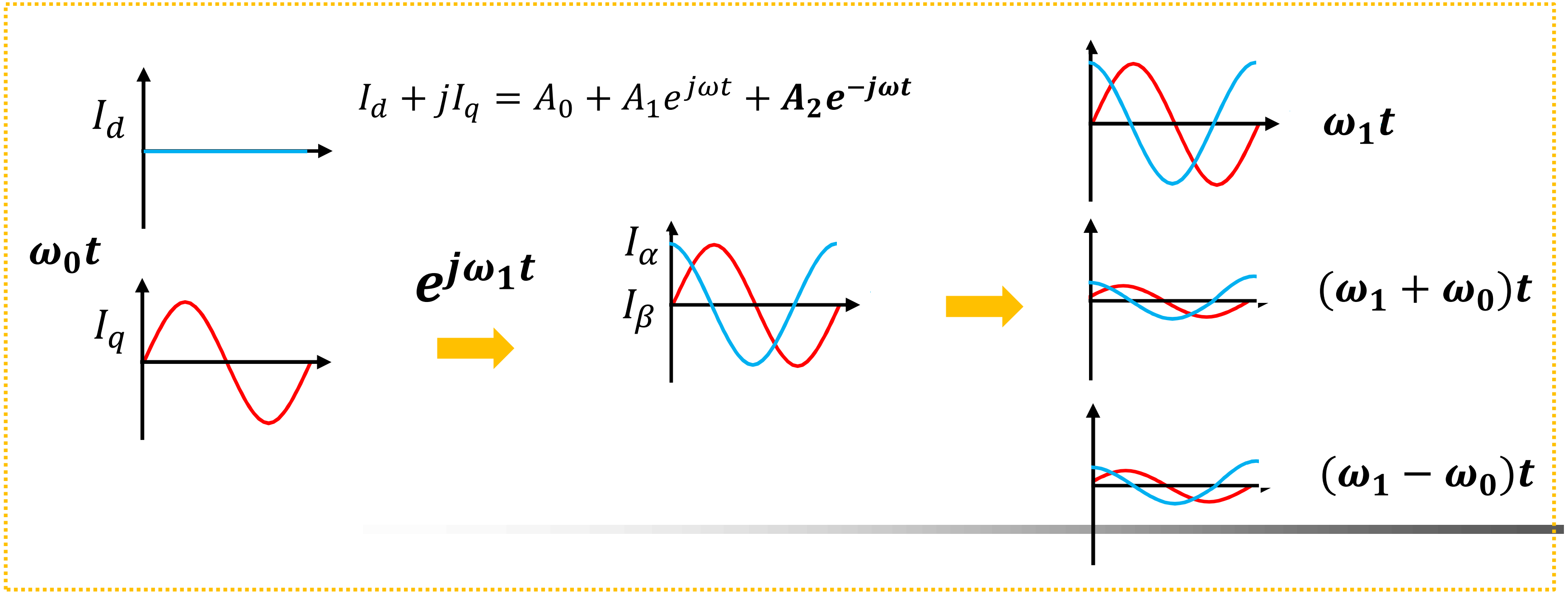
Harmonic amplification and super-synchronous sub-synchronous

$$v_D\cdot\cos(\theta_o) - v_Q\cos(\theta_o) \qquad v_D\cdot\sin(\theta_o) + v_Q\cos(\theta_o)$$

$$= V\sin(\theta_o) \qquad = V\cos(\theta_o)$$

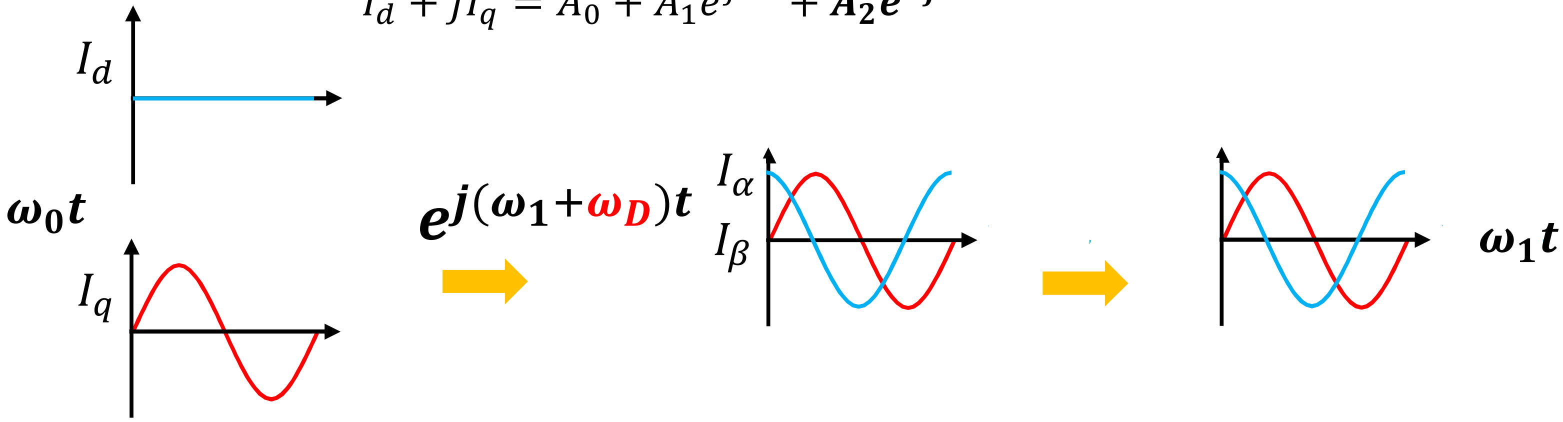
3-Phase Locked Loop (PLL) - D/Q control effect - $v_q = 0$ and $I_q = 0$

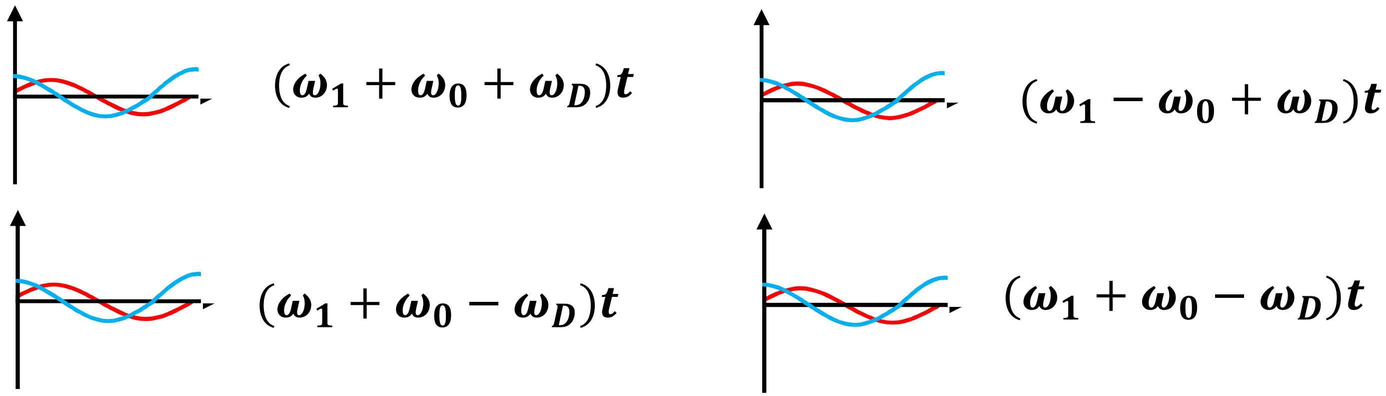




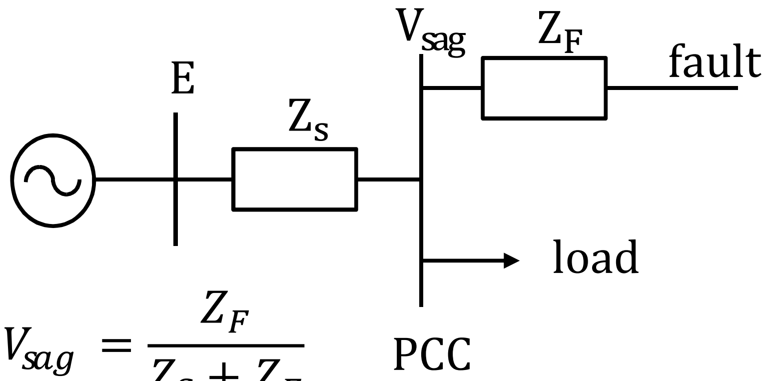
3-Phase Locked Loop (PLL) - D/Q control effect - $v_q = 0$

$$I_d + jI_q = A_0 + A_1 e^{j\omega t} + \boldsymbol{A_2 e^{-j\omega t}}$$





Voltage Sag Type A, B, C, D, E, F, G



$$V_{sag} = \frac{Z_F}{Z_S + Z_F}$$

- **Fault Type**
  - Single Phase Ground Fault (SLG)
  - Phase-Phase Fault (LL)
  - Double Phase Ground Fault(LLG)
  - 3 Phase Fault (3P)
- **Transformer Winding**
  - Type 1 : Yn-Yn
  - Type 2 **: without Zero sequence**
    Y-Y(no ground), Δ-Δ, D-Zig
  - Type 3 : D-Y, Y-D, Y-Zig
- **Load Connection**
  - Y-Connected
  - Delta-Connected

Voltage Sag Type	Fault Type				Transformer Type			Load Connection	
	3P	LL	SLG	LLG	1	2	3	Wye	Delta
A	■							■	
	■								■
	■				■			■	
	■					■		■	
	■						■	■	
	■				■				■
	■					■			■
	■						■		■
B			■					■	
			■		■			■	
C			■						■
			■		■				■
			■			■			■
			■				■	■	
		■						■	
		■			■			■	
		■				■		■	
		■					■		■
D			■			■		■	
			■				■		■
		■							■
		■			■				■
		■				■			■
		■					■	■	
E				■				■	
				■	■			■	
F				■					■
				■	■				■
				■			■	■	
G				■		■		■	
				■			■		■

Sag Type A

$$V_a = hV$$
$$V_b = -\frac{1}{2}hV - j\frac{\sqrt{3}}{2}hV$$
$$V_c = -\frac{1}{2}hV + j\frac{\sqrt{3}}{2}hV$$

Sag Type B

$$V_a = hV$$
$$V_b = -\frac{1}{2}V - j\frac{\sqrt{3}}{2}V$$
$$V_c = -\frac{1}{2}V + j\frac{\sqrt{3}}{2}V$$

Sag Type C

$$V_a = hV$$
$$V_b = -\frac{1}{2}V - j\frac{\sqrt{3}}{2}hV$$
$$V_c = -\frac{1}{2}V + j\frac{\sqrt{3}}{2}hV$$

Sag Type D

$$V_a = hV$$
$$V_b = -\frac{1}{2}hV - j\frac{\sqrt{3}}{2}V$$
$$V_c = -\frac{1}{2}hV + j\frac{\sqrt{3}}{2}V$$

Sag Type E

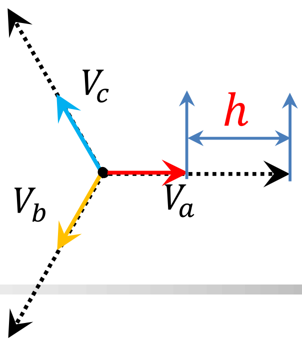
$$V_a = V$$
$$V_b = -\frac{1}{2}hV - j\frac{\sqrt{3}}{2}V$$
$$V_c = -\frac{1}{2}hV + j\frac{\sqrt{3}}{2}V$$

Sag Type F

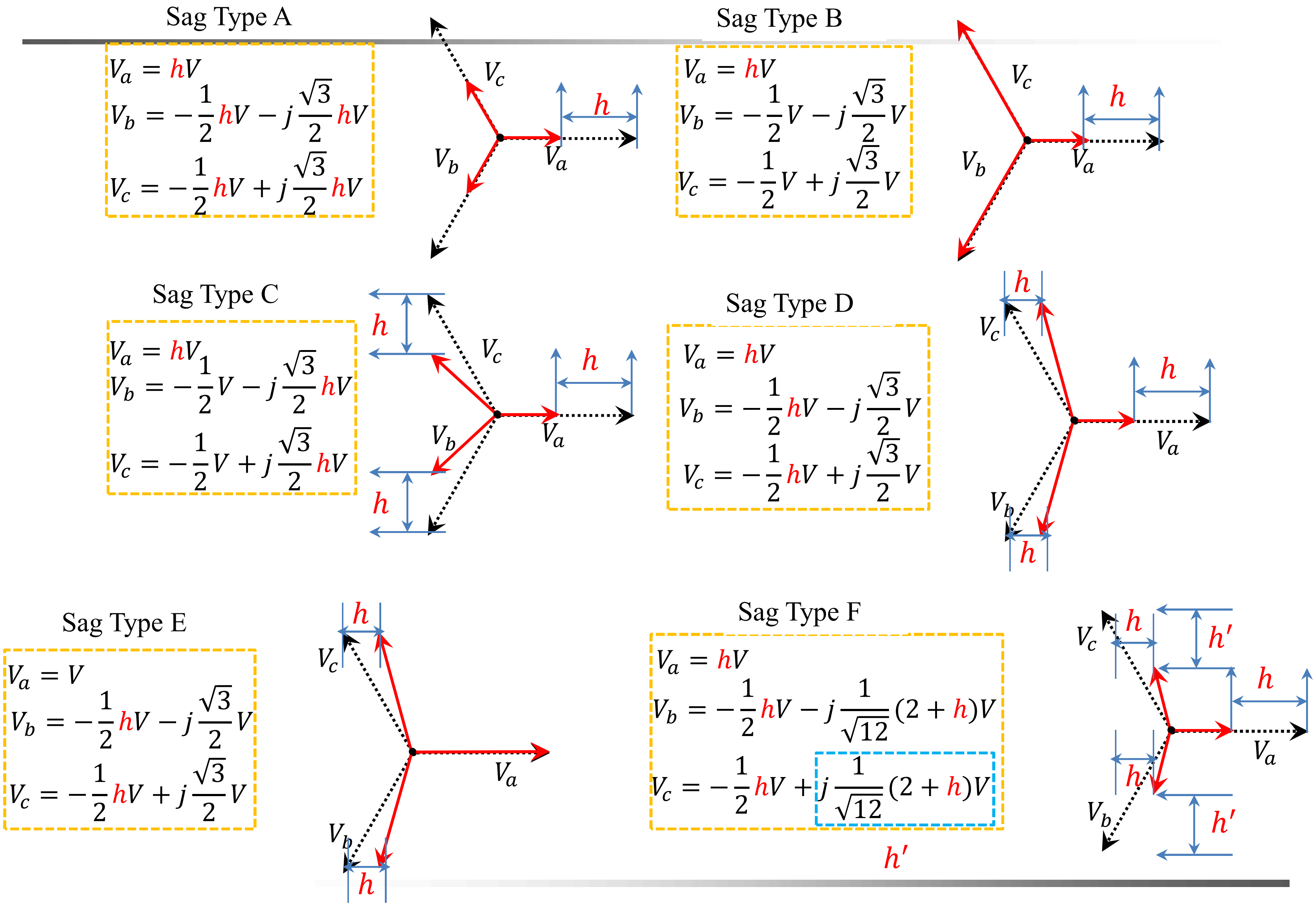
$$V_a = hV$$
$$V_b = -\frac{1}{2}hV - j\frac{1}{\sqrt{12}}(2+h)V$$
$$V_c = -\frac{1}{2}hV + j\frac{1}{\sqrt{12}}(2+h)V$$

Sag Type G

$$V_a = \frac{1}{3}(2+h)V$$
$$V_b = -\frac{1}{6}(2+h)V - j\frac{\sqrt{3}}{2}hV$$
$$V_c = -\frac{1}{6}(2+h)V + j\frac{\sqrt{3}}{2}hV$$



$$\begin{cases} h : 0.1 \sim 0.9 \ (Sag\ depth) \\ V = \dfrac{V_{LN}}{\sqrt{3}} : Phase\ to\ ground \\ V_{a,b,c} : Sag\ voltage \end{cases}$$



Sag Type A
$V_a = hV$
$V_b = -\frac{1}{2}hV - j\frac{\sqrt{3}}{2}hV$
$V_c = -\frac{1}{2}hV + j\frac{\sqrt{3}}{2}hV$
V_a V_b V_c h
Sag Type B
$V_a = hV$
$V_b = -\frac{1}{2}V - j\frac{\sqrt{3}}{2}V$
$V_c = -\frac{1}{2}V + j\frac{\sqrt{3}}{2}V$
V_a V_b V_c h
Sag Type C
$V_a = hV$
$V_b = -\frac{1}{2}V - j\frac{\sqrt{3}}{2}hV$
$V_c = -\frac{1}{2}V + j\frac{\sqrt{3}}{2}hV$
V_a V_b V_c h
Sag Type D
$V_a = hV$
$V_b = -\frac{1}{2}hV - j\frac{\sqrt{3}}{2}V$
$V_c = -\frac{1}{2}hV + j\frac{\sqrt{3}}{2}V$
V_a V_b V_c h
Sag Type E
$V_a = V$
$V_b = -\frac{1}{2}hV - j\frac{\sqrt{3}}{2}V$
$V_c = -\frac{1}{2}hV + j\frac{\sqrt{3}}{2}V$
V_a V_b V_c h
Sag Type F
$V_a = hV$
$V_b = -\frac{1}{2}hV - j\frac{1}{\sqrt{12}}(2+h)V$
$V_c = -\frac{1}{2}hV + j\frac{1}{\sqrt{12}}(2+h)V$
h'
V_a V_b V_c h h'

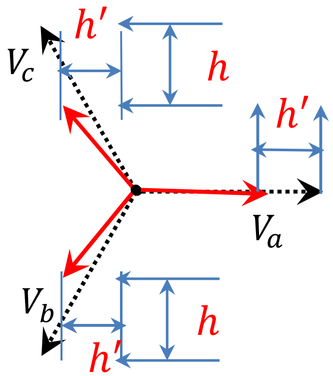
Sag Type G

$$V_a = \frac{1}{3}(2+h)V$$

$$V_b = -\frac{1}{6}(2+h)V - j\frac{\sqrt{3}}{2}hV$$

$$V_c = -\frac{1}{6}(2+h)V + j\frac{\sqrt{3}}{2}hV$$

h'

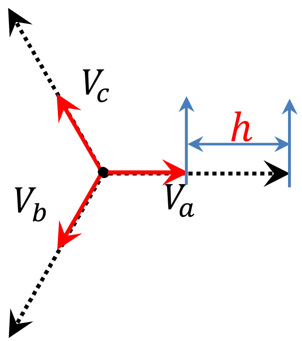


Sag Type A

$$V_a = hV$$

$$V_b = -\frac{1}{2}hV - j\frac{\sqrt{3}}{2}hV$$

$$V_c = -\frac{1}{2}hV + j\frac{\sqrt{3}}{2}hV$$



Sag Type A

$$V_P = hV$$

$$V_N = 0$$

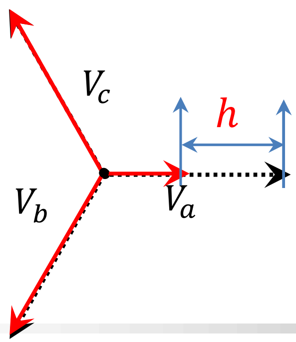
3-phase fault

Sag Type B

$$V_a = hV$$

$$V_b = -\frac{1}{2}V - j\frac{\sqrt{3}}{2}V$$

$$V_c = -\frac{1}{2}V + j\frac{\sqrt{3}}{2}V$$



Sag Type B

$$V_P = \frac{h+2}{3}V$$

$$V_N = -\frac{(1-h)}{3}V$$

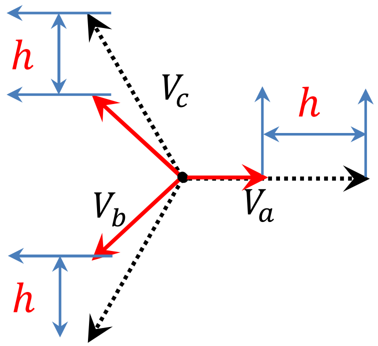
Single-phase fault
(with ground point)

Sag Type C

$$V_a = hV$$

$$V_b = -\frac{1}{2}V - j\frac{\sqrt{3}}{2}hV$$

$$V_c = -\frac{1}{2}V + j\frac{\sqrt{3}}{2}hV$$



$$V_P = \frac{1+h}{2}V$$

$$V_N = \frac{(1-h)}{2}V$$

Single-phase fault (with ground point)

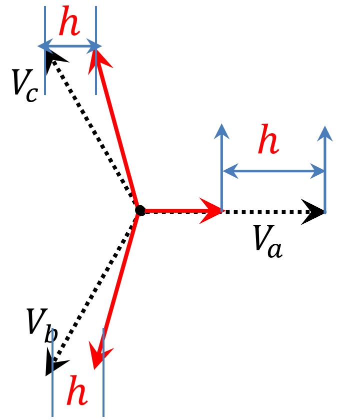
Double-phase fault (with ground point)

Sag Type D

$$V_a = hV$$

$$V_b = -\frac{1}{2}hV - j\frac{\sqrt{3}}{2}V$$

$$V_c = -\frac{1}{2}hV + j\frac{\sqrt{3}}{2}V$$



$$V_P = \frac{1+h}{2}V$$

$$V_N = -\frac{(1-h)}{2}V$$

Single-phase fault (with ground point)

Double-phase fault (with ground point)

Sag Type A

$$V_P = hV$$
$$V_N = 0$$

$$V_P = |V_P|e^{j\omega t} \quad V_N = |V_N|e^{-j\omega t}$$

$$r_{max} = |V_P| + |V_N|$$

$$r_{min} = |V_P| - |V_N|$$

$$\varphi_{inc} = \frac{1}{2}(\varphi_P + \varphi_N)$$

$$SI = \frac{r_{min}}{r_{max}} \begin{cases} SI = 1 : \text{Circle} \\ 0 < SI < 1 : \text{Ellipse} \\ \mathbf{SI = 0 : Straight\ line} \end{cases}$$

Sag Type B

$$V_P = \frac{h+2}{3}V$$
$$V_N = -\frac{(1-h)}{3}V$$
$$V_0 = -\frac{1-h}{3}V$$

Sag Type C

$$V_P = \frac{1+h}{2}V$$
$$V_N = \frac{(1-h)}{2}V$$

Sag Type E

$$V_P = \frac{1+2h}{3}V$$
$$V_N = \frac{(1-h)}{3}V$$
$$V_0 = \frac{1-h}{3}V$$

Sag Type D

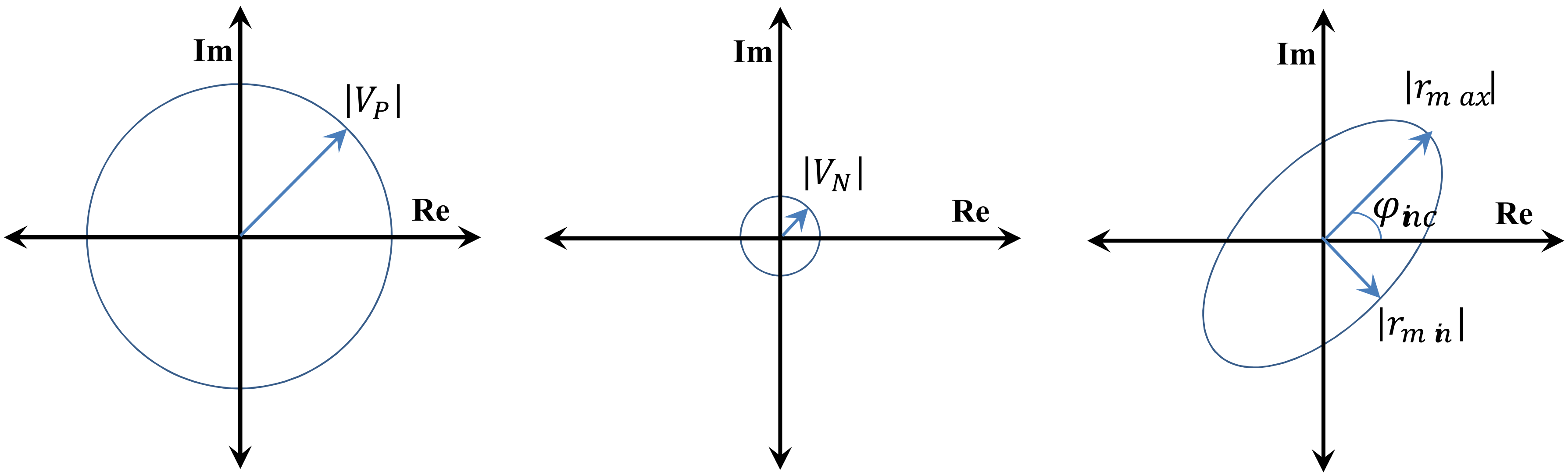
$$V_P = \frac{1+h}{2}V$$
$$V_N = -\frac{(1-h)}{2}V$$

Sag Type F

$$V_P = \frac{1+2h}{3}V$$
$$V_N = -\frac{(1-h)}{3}V$$

Sag Type G

$$V_P = \frac{1+2h}{3}V$$
$$V_N = \frac{(1-h)}{3}V$$



Singularity Instability

Single Phase Voltage Dip Characteristics

Type	Space vector				Zero sequence voltage
	SI	φ_{inc}	r_{min}	r_{max}	
B	$1-\frac{2}{3}d$	$\frac{5\pi}{6}-n\frac{\pi}{3}$	$\left(1-\frac{2}{3}d\right)V$	V	$-\frac{d}{3}V\cos\left(\omega t+\varphi-(n-1)\frac{2\pi}{3}\right)$
D	$1-d$	$\frac{5\pi}{6}-n\frac{\pi}{3}$	$(1-d)V$	V	0
F	$\frac{3(1-d)}{3-d}$	$\frac{5\pi}{6}-n\frac{\pi}{3}$	$(1-d)V$	$\left(1-\frac{d}{3}\right)V$	0

Double Phase Voltage Dip Characteristics

Type	Space vector				Zero sequence voltage
	SI	φ_{inc}	r_{min}	r_{max}	
C	$1-d$	$(1-n)\frac{\pi}{3}$	$(1-d)V$	V	0
E	$\frac{3(1-d)}{3-d}$	$(1-n)\frac{\pi}{3}$	$(1-d)V$	$\left(1-\frac{d}{3}\right)V$	$\frac{d}{3}V\cos\left(\omega t+\varphi-(n-1)\frac{2\pi}{3}\right)$
G	$\frac{3(1-d)}{3-d}$	$(1-n)\frac{\pi}{3}$	$(1-d)V$	$\left(1-\frac{d}{3}\right)V$	0

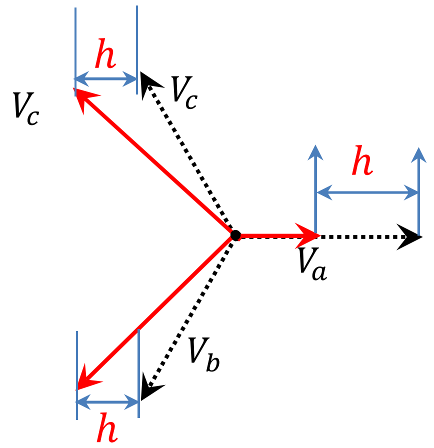
n : 1,2,3 (a phase, b phase, c phase)

Singularity Instability

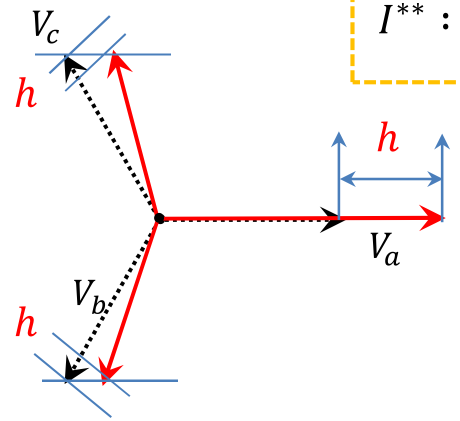
n : 1,2,3 (a phase, b phase, c phase)

Type	Space vector				Zero sequence voltage
	SI	φ_{inc}	r_{min}	r_{max}	
H	1	-	V	V	$-dV\cos\left(\omega t+\varphi-(n-1)\frac{2\pi}{3}\right)$
I*	1	-	V	V	$2dV\cos\left(\omega t+\varphi-(n-1)\frac{2\pi}{3}\right)$
I**	$\frac{4(1-d)}{3}$	$(1-n)\frac{\pi}{3}$	$\frac{4}{3}(1-d)V$	V	$\frac{1}{2}V\cos\left(\omega t+\varphi-(n-1)\frac{2\pi}{3}\right)$

Swell Type H



Swell Type I



$I^{*}: 0 < h < \frac{1}{4}V$

$I^{**}: \frac{1}{4}V < h < V$

Singularity Instability

AC Network Faults → ***Positive Sequence = Negative Sequence*** ▶ ***Unstable!***

Singularity Instability → ***"v_α or v_β is Zero"***

$$\begin{bmatrix} P \\ Q \\ P_0 \end{bmatrix} = \begin{bmatrix} \bar{P} + P_{c2} \cos(2\omega t) + P_{s2} \sin(2\omega t) \\ \bar{Q} + Q_{c2} \cos(2\omega t) + Q_{s2} \sin(2\omega t) \\ \bar{P}_0 + P_{0c2} \cos(2\omega t) \end{bmatrix}$$

$$\begin{bmatrix} \bar{P} \\ P_{c2} \\ P_{s2} \\ \bar{Q} \\ Q_{c2} \\ Q_{s2} \end{bmatrix} = \frac{3}{2} \begin{bmatrix} V_{sd}^{+} & V_{sq}^{+} & V_{sd}^{-} & V_{sq}^{-} \\ V_{sd}^{-} & V_{sq}^{-} & V_{sd}^{+} & V_{sq}^{+} \\ V_{sq}^{-} & -V_{sd}^{-} & -V_{sq}^{+} & V_{sd}^{+} \\ V_{sq}^{+} & -V_{sd}^{+} & V_{sq}^{-} & -V_{sd}^{-} \\ V_{sq}^{-} & -V_{sd}^{-} & V_{sq}^{+} & -V_{sd}^{+} \\ -V_{sd}^{-} & -V_{sq}^{-} & V_{sd}^{+} & V_{sq}^{+} \end{bmatrix} \begin{bmatrix} i_d^{+} \\ i_q^{+} \\ i_d^{-} \\ i_q^{-} \end{bmatrix}$$

$$i_d^{+\,ref} = \frac{2V_{sd}^{+} P_s^{ref}}{3[(V_{sd}^{+})^2 - (V_{sd}^{-})^2]}$$

$$i_q^{+\,ref} = -\frac{2V_{sd}^{+} Q_s^{ref}}{3[(V_{sd}^{+})^2 + (V_{sd}^{-})^2]}$$

$$i_d^{-\,ref} = \frac{2V_{sd}^{-} P_s^{ref}}{3[(V_{sd}^{+})^2 - (V_{sd}^{-})^2]}$$

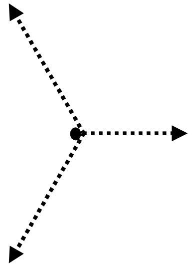
$$i_q^{-ref} = -\frac{2V_{sd}^{-} Q_s^{ref}}{3[(V_{sd}^{+})^2 + (V_{sd}^{-})^2]}$$

Singularity Instability Condition (Positive sequence = Negative sequence)

Sag Type A

$$\underline{V}_g^a = 0 \qquad \underline{V}_g^b = 0 \qquad \underline{V}_g^c = 0$$

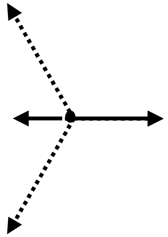
$$\underline{V}_g^+ = 0 \qquad \underline{V}_g^- = 0 \qquad \underline{V}_g^0 = 0$$



Sag Type C

$$\underline{V}_g^a = E_1 \qquad \underline{V}_g^b = -\frac{1}{2}E_1 \qquad \underline{V}_g^c = -\frac{1}{2}E_1$$

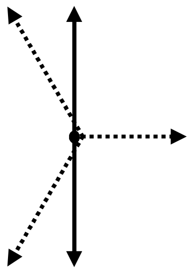
$$\underline{V}_g^+ = \frac{1}{2}E_1 \qquad \underline{V}_g^- = \frac{1}{2}E_1 \qquad \underline{V}_g^0 = 0$$



Sag Type D

$$\underline{V}_g^a = 0 \qquad \underline{V}_g^b = -\frac{1}{2}jE_1\sqrt{3} \qquad \underline{V}_g^c = +\frac{1}{2}jE_1\sqrt{3}$$

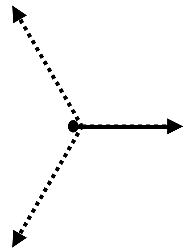
$$\underline{V}_g^+ = \frac{1}{2}E_1 \qquad \underline{V}_g^- = -\frac{1}{2}E_1 \qquad \underline{V}_g^0 = 0$$



Sag Type E

$$\underline{V}_g^a = E_1 \qquad \underline{V}_g^b = 0 \qquad \underline{V}_g^c = 0$$

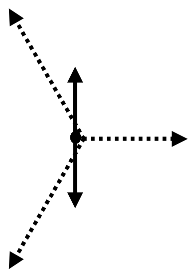
$$\underline{V}_g^+ = \frac{1}{2}E_1 \qquad \underline{V}_g^- = -\frac{1}{2}E_1 \qquad \underline{V}_g^0 = 0$$



Sag Type F

$$\underline{V}_g^a = 0 \qquad \underline{V}_g^b = j\frac{\sqrt{3}}{3}E_1 \qquad \underline{V}_g^c = j\frac{\sqrt{3}}{3}E_1$$

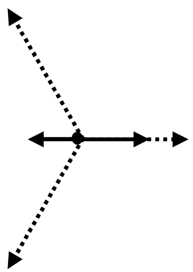
$$\underline{V}_g^+ = \frac{1}{3}E_1 \qquad \underline{V}_g^- = -\frac{1}{3}E_1 \qquad \underline{V}_g^0 = 0$$



Sag Type G

$$\underline{V}_g^a = \frac{2}{3}E_1 \qquad \underline{V}_g^b = -\frac{1}{3}E_1 \qquad \underline{V}_g^c = -\frac{1}{3}E_1$$

$$\underline{V}_g^+ = \frac{1}{3}E_1 \qquad \underline{V}_g^- = \frac{1}{3}E_1 \qquad \underline{V}_g^0 = 0$$



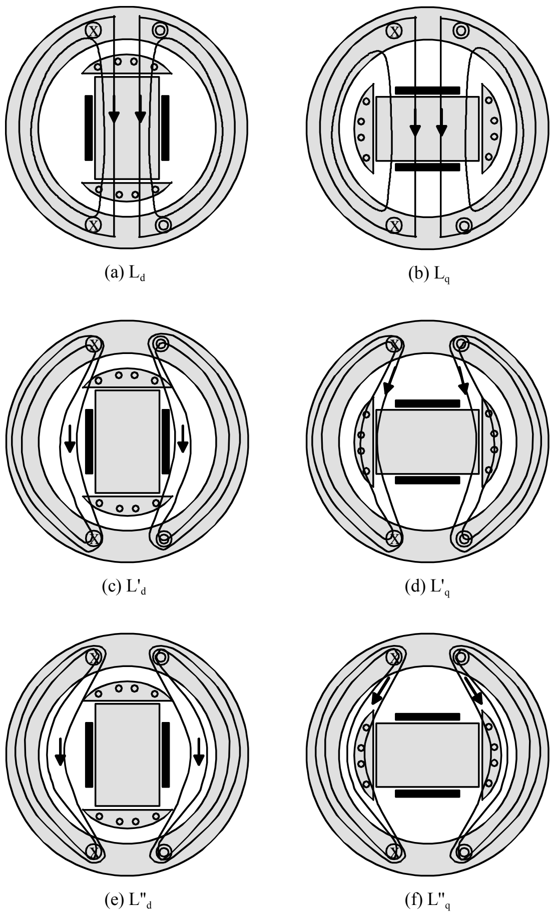
II. SCR (Short Circuit Ratio)

SCR (Short Circuit Ratio)

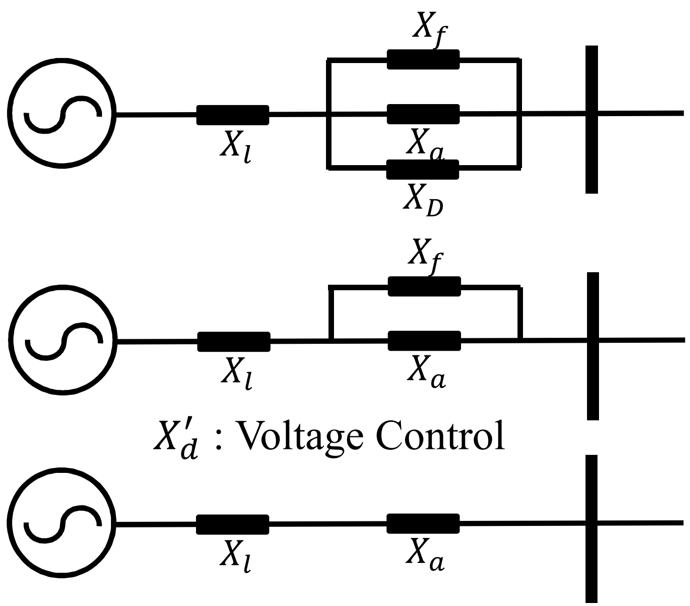
$$Short\ Circuit\ Ratio = \frac{I_f\ for\ Open\ Circuit\ Voltage}{I_f\ for\ Short\ Circuit\ Current} = \frac{1}{X_d[\Omega]}$$

$$\approx Reluctance\ of\ air\ gap \approx \frac{1}{L_s} : Synchronous\ inductance$$

Generator



X'_d : Protection Relay



X_l : leakage inductance
X_D : damping inductance
X_f : field inductance
X_a : armature inductance

$$P_G = \frac{E_1 E_2 \sin\theta}{X_d}$$

Operating Characteristics,
Physical size,
Cost of machine

After Faults

* Flux Change

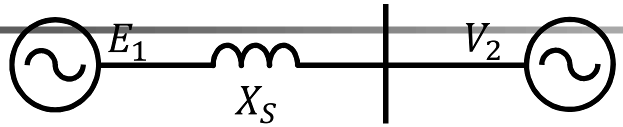
* Inductance Change

* Time Constant Change

* Sub-transient Current

* Transient Current

* Steady-state Current



$$V_2 = E_1 - jX_s I \qquad S = P_{bad} + jQ_{bad} = V_2 I^*$$

$$S = V_2 \frac{E_1^* - V_2^*}{-jX_s} = \frac{j}{X_s}(E_1 V_2 \cos\delta + jE_1 V_2 \sin\delta - V_2^2)$$

$$P_2 = -\frac{E_1 V_2}{X_s}\sin\delta \qquad Q_2 = -\frac{V_2^2}{X_s} + \frac{E_1 V_2}{X_s}\cos\delta$$

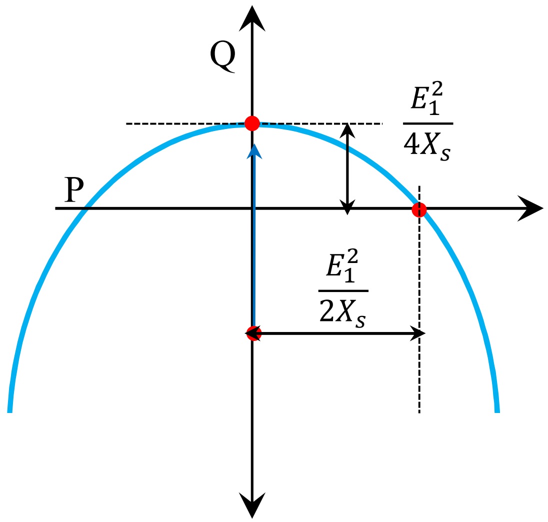
$$(V_2^2)^2 + (2QX_s - E_1^2)V_2^2 + X_s^2(P^2 + Q^2) = 0$$

$$(2QX_s - E_1^2)^2 - 4X_s^2(P^2 + Q^2) \geq 0$$

$$-P^2 - \frac{E_1^2}{X_s}Q + \left(\frac{E_1^2}{2X_s}\right)^2 \geq 0$$

$$P\Big|_{Q=0} \leq \frac{E_1^2}{2X_s} \qquad Q\Big|_{P=0} \leq \frac{E_1^2}{4X_s}$$

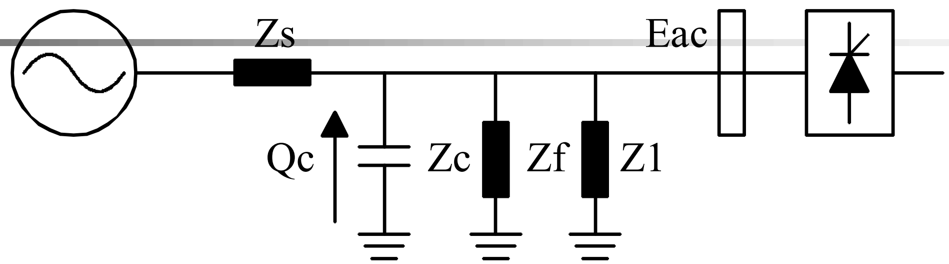
$$V_2 = \sqrt{\frac{E_1^2}{2} - QX_s \pm \sqrt{\frac{E_1^4}{4} - X_s^2 P^2 - X_s E_1^2 Q}}$$



$$P_{max} = \frac{\cos\emptyset}{1 + \sin\emptyset} \cdot \frac{E_1^2}{2X_s}$$

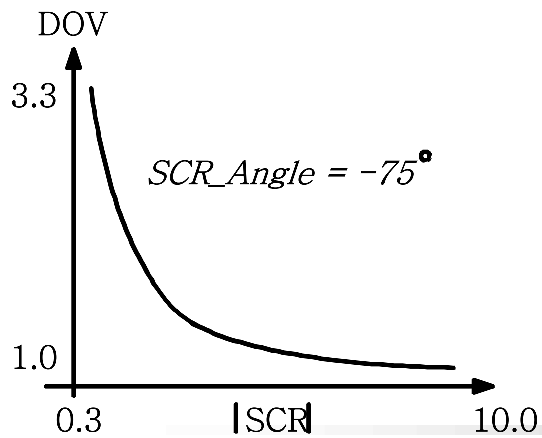
$$Q_{max} = \frac{\sin\emptyset}{1 + \sin\emptyset} \cdot \frac{E_1^2}{2X_s}$$

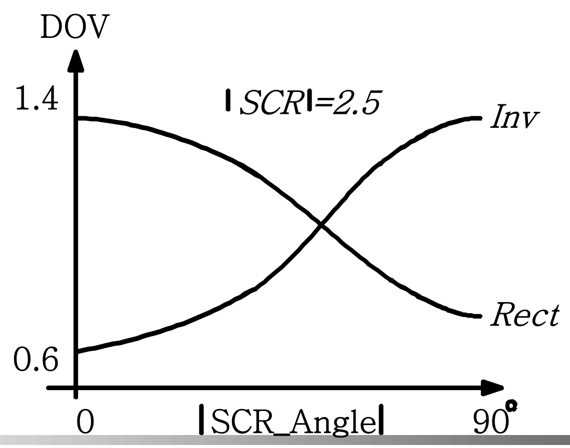
$$V_{max} = \frac{1}{\sqrt{1 + \sin\emptyset}} \cdot \frac{E_1}{\sqrt{2}}$$

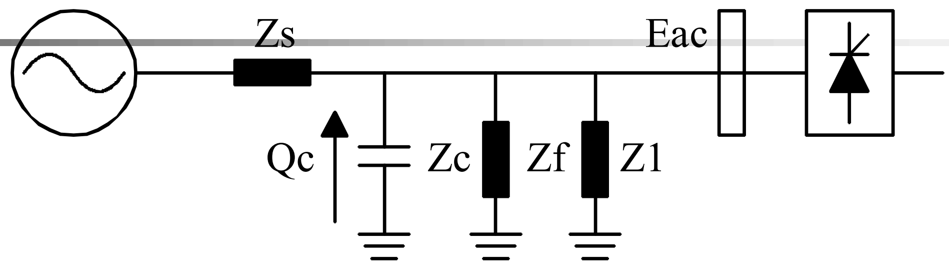


$$SCL = \frac{E_{ac}^2}{Z_{ac}} \qquad SCR = \frac{SCL\ (Short\ Circuit\ Ratio\ (MVA))}{P_{DC}(DC\ power\ (MW))}$$

$$SCR = \frac{MVA(SCL)}{P_{DC}} = \frac{E_{ac}^2}{P_{DC} \cdot Z_{ac}} = \frac{1}{Z_{ac}} \cdot Z_{base} = (\frac{1}{Z_s} + \frac{1}{Z_l}) \cdot Z_{base}$$

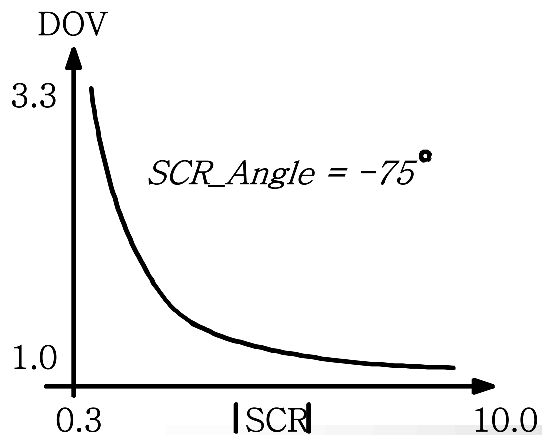


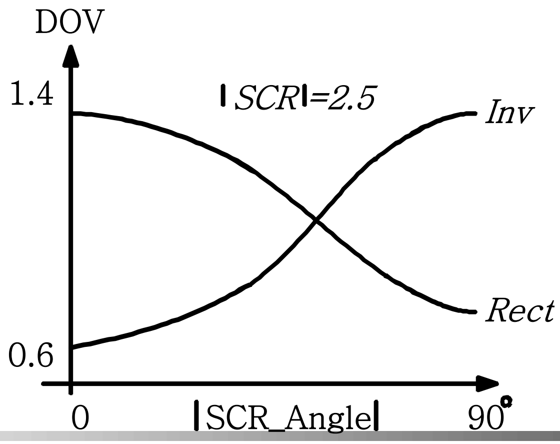




$$SCL = \frac{E_{ac}^2}{Z_{ac}} \qquad SCR = \frac{SCL\ (Short\ Circuit\ Ratio\ (MVA))}{P_{DC}(DC\ power\ (MW))}$$

$$SCR = \frac{MVA(SCL)}{P_{DC}} = \frac{E_{ac}^2}{P_{DC} \cdot Z_{ac}} = \frac{1}{Z_{ac}} \cdot Z_{base} = \left(\frac{1}{Z_s} + \frac{1}{Z_l}\right) \cdot Z_{base}$$



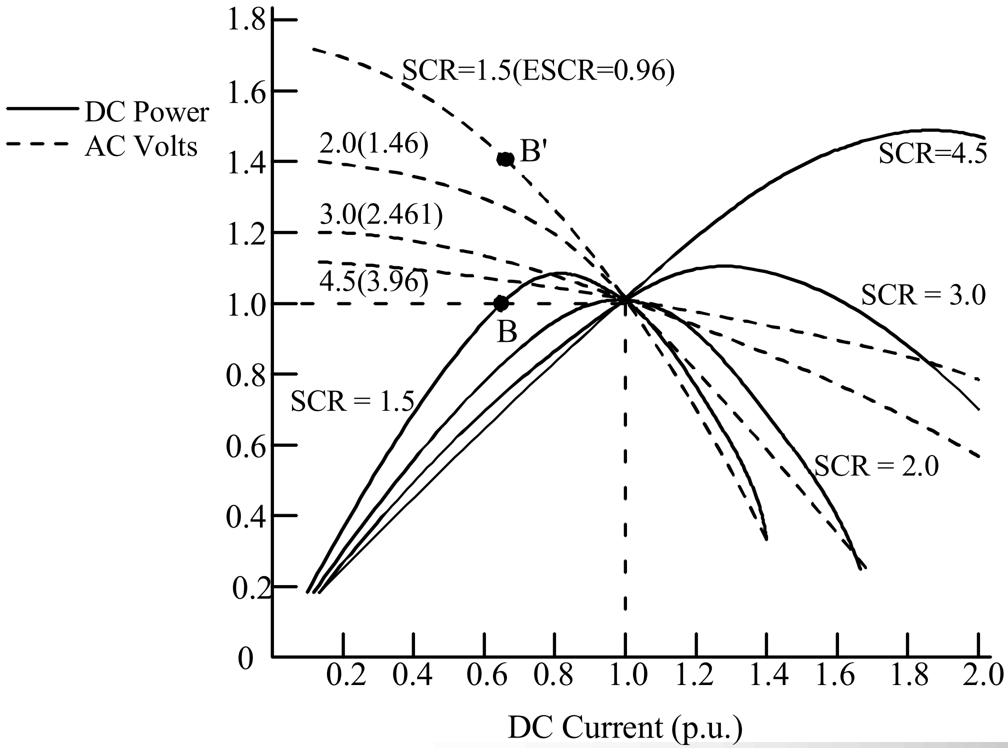


$$Q_C = \frac{E_{ac}^2}{\frac{1}{Z_c} + \frac{1}{Z_f}} \qquad ESCR = \frac{SCL - Q_C}{P_{DC}} = \frac{1}{Z_e} \cdot Z_{base} = \left(\frac{1}{Z_s} + \frac{1}{Z_c} + \frac{1}{Z_f} + \frac{1}{Z_l}\right) \cdot Z_{base}$$

$$RESCR = \frac{SCL - Q_C}{P_{DC} + Q_d}$$

CSCR(Critical Short Circuit Ratio), CESCR

Q_d : Reactive power consumption of converter



X_C=0.15 p.u., γ =18°

Q_C=Q_d=0.54Pdn at V_L=1.0 p.u.

MAP
(Maximum Available Power)

$$L = \frac{Z_{ac}}{\omega_0} = \frac{E_{ac}^2}{\omega_0 \cdot (SCR \cdot P_{DC})}$$

$$\omega_{reson} = \frac{1}{\sqrt{L \cdot C}} = \omega_0 \sqrt{\frac{SCR}{0.5 \sim 0.6}}$$

$$\omega_{reson} = \omega_0 \sqrt{\frac{2.5}{0.6}} = 2\omega_0$$

WSCR(Weighted SCR)

$$WSCR = \frac{Weighted\ SCC_{MVA}}{\sum_i^n P_{MW\,i}} = \frac{\frac{\sum_i^n SCC_{MVAi}\cdot P_{MW\,i}}{\sum_i^n P_{MW\,i}}}{\sum_i^n P_{MW\,i}} = \frac{\sum_i^n SCC_{MVAi}\cdot P_{MW\,i}}{(\sum_i^n P_{MW\,i})^2}$$

CSCR(Composite SCR)

$$CSCR = \frac{SCC_{MVA}}{\sum_i^n P_{MW\,i}}$$

Site Dependent SCR

$$SDSCR = \frac{\left|v_{R,i}\right|^2}{(P_{R,i} + \sum_{j\in R, j\neq i}^{j} P_{R,j}\cdot w_{ij})}, w_{i,j} = \frac{Z_{RR,i}}{Z_{RR,i}}\left(\frac{V_{R,j}}{V_{R,j}}\right)$$

SCR with Interaction Factor

$$SCRIF = \frac{SCC_i}{P_i + \sum_i^j (IF_{ji}\cdot P_j)}, IF_{ji} = \frac{\Delta V_i}{\Delta V_j}$$

Inverter Interaction Level SCR

$$IILSCR = \frac{SCC_{MVAi}}{P_{IBRi} + \sum_{m=1,m\neq i}^{N} P_{IBR\,(m-1)}}$$

Multi-Infeed Effective SCR

$$MIESCR = \frac{SCC_i}{P_{DC,i} + \sum_{j=1,j\neq i}^{k}(MIIF_{ji}\cdot P_{DCj})}, MIIF_{ji} = \frac{V_i}{V_j} = \frac{Z_{ij}}{Z_{jj}}$$

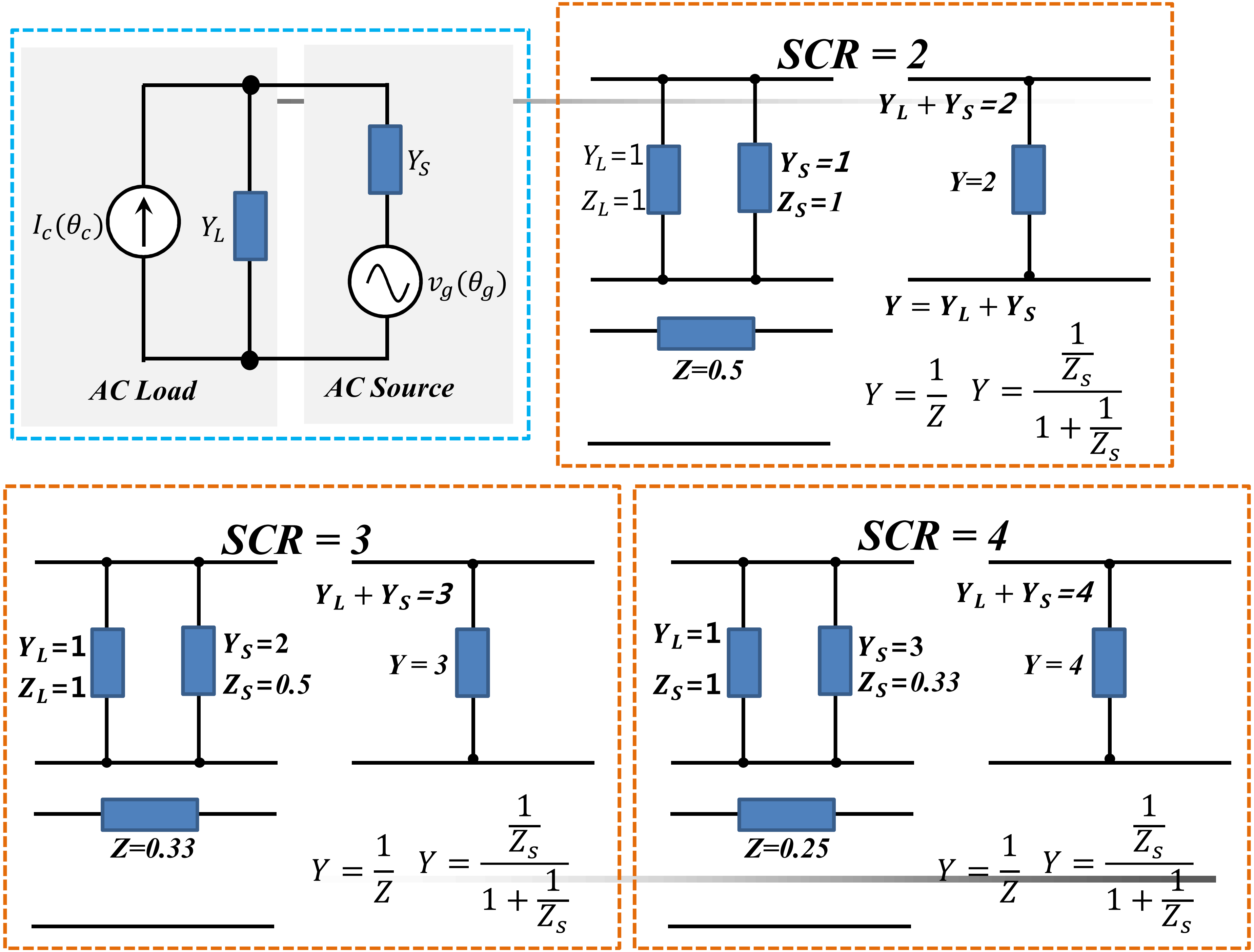
$$SCR = \frac{AC}{DC} = \frac{Load}{Supply} = \frac{1}{X_s}$$

SCR = 4

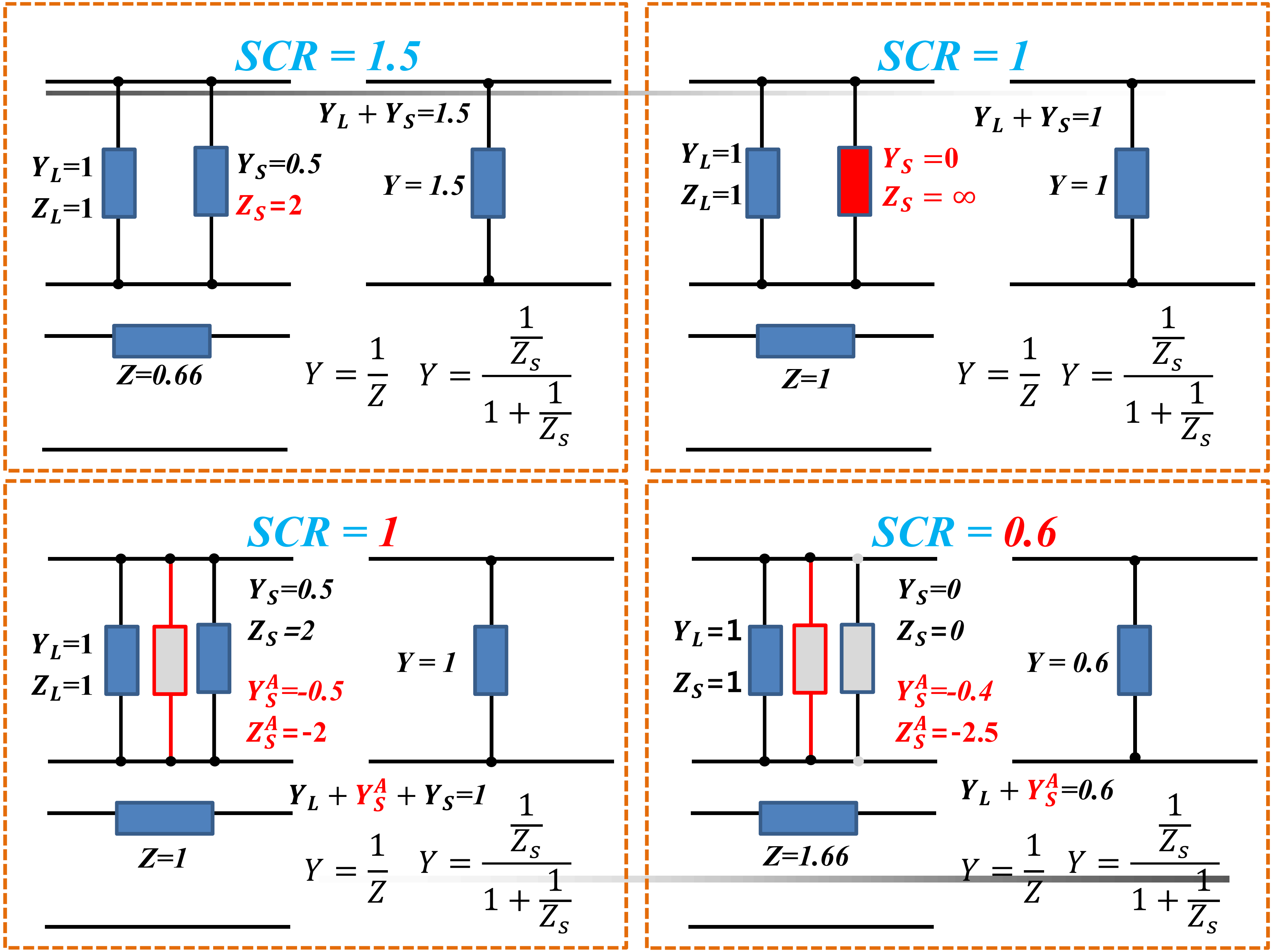
SCR = 2

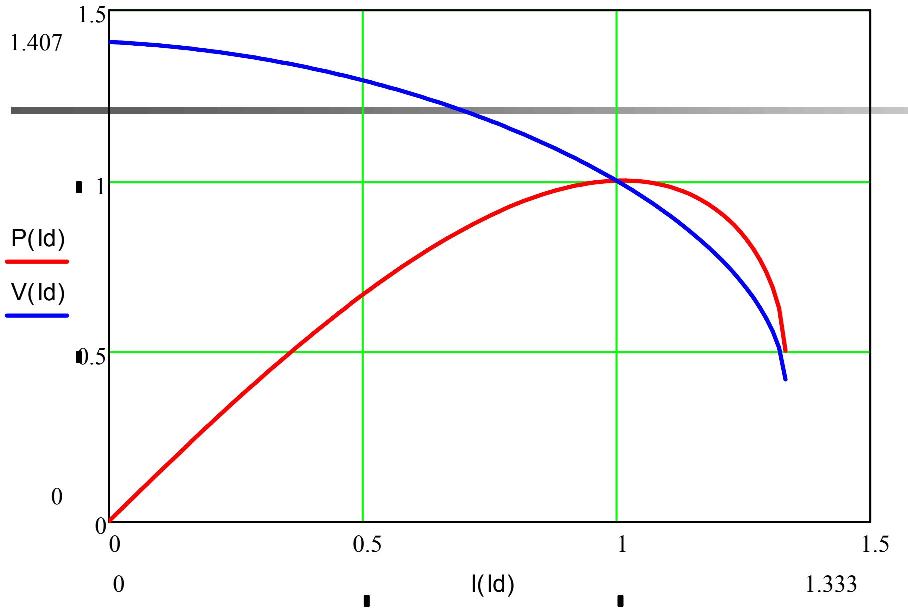
SCR = 1.5

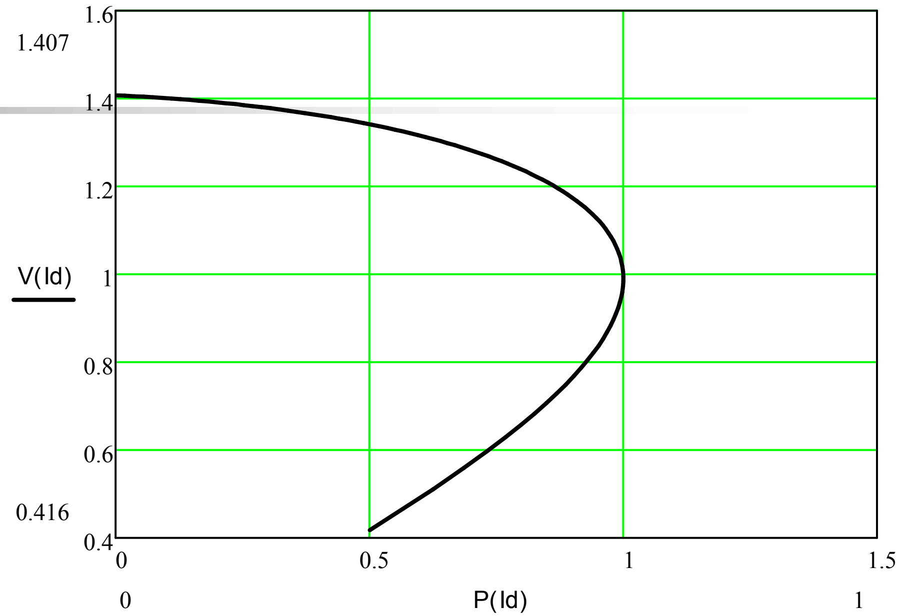
Voltage Stability

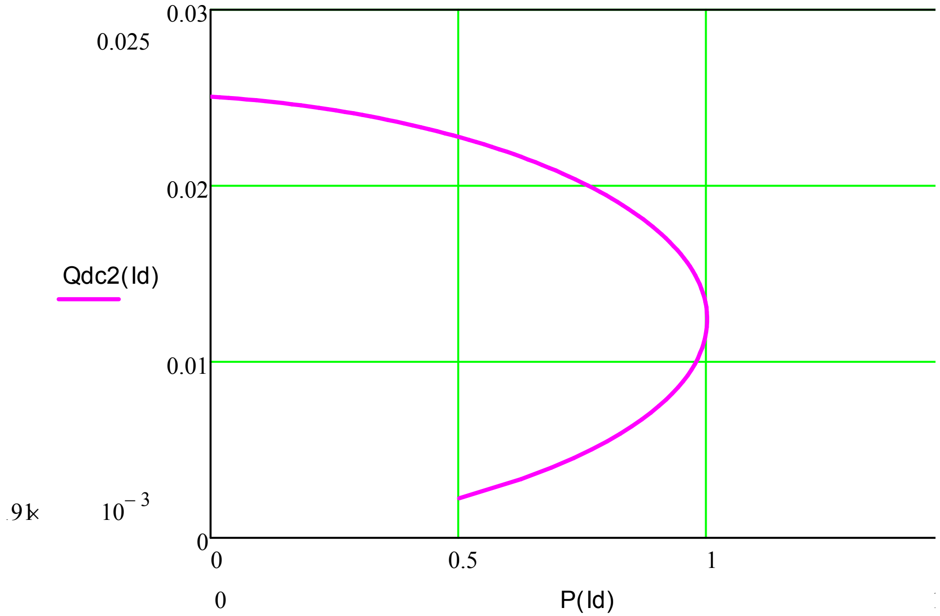


$I_c(\theta_c)$
Y_L
Y_S
$v_g(\theta_g)$
AC Load
AC Source
SCR = 2
$Y_L=1$
$Z_L=1$
$Y_S=1$
$Z_S=1$
$Y_L+Y_S=2$
$Y=2$
$Y=Y_L+Y_S$
$Z=0.5$
$Y=\frac{1}{Z}$
$Y=\frac{\frac{1}{Z_S}}{1+\frac{1}{Z_S}}$
SCR = 3
$Y_L=1$
$Z_L=1$
$Y_S=2$
$Z_S=0.5$
$Y_L+Y_S=3$
$Y=3$
$Z=0.33$
$Y=\frac{1}{Z}$
$Y=\frac{\frac{1}{Z_S}}{1+\frac{1}{Z_S}}$
SCR = 4
$Y_L=1$
$Z_S=1$
$Y_S=3$
$Z_S=0.33$
$Y_L+Y_S=4$
$Y=4$
$Z=0.25$
$Y=\frac{1}{Z}$
$Y=\frac{\frac{1}{Z_S}}{1+\frac{1}{Z_S}}$

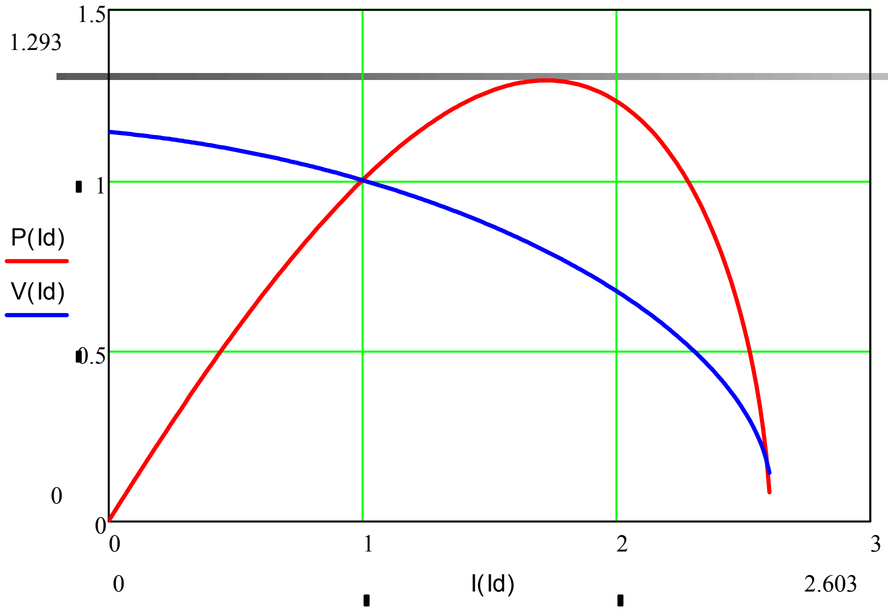

SCR = 1.5
Y_L + Y_S=1.5
Y_L=1
Z_L=1
Y_S=0.5
Z_S=2
Y = 1.5
Z=0.66
Y = 1/Z
Y = (1/Z_S)/(1 + 1/Z_S)
SCR = 1
Y_L + Y_S=1
Y_L=1
Z_L=1
Y_S =0
Z_S = ∞
Y = 1
Z=1
Y = 1/Z
Y = (1/Z_S)/(1 + 1/Z_S)
SCR = 1
Y_L=1
Z_L=1
Y_S=0.5
Z_S =2
Y_S^A=-0.5
Z_S^A=-2
Y = 1
Y_L + Y_S^A + Y_S=1
Z=1
Y = 1/Z
Y = (1/Z_S)/(1 + 1/Z_S)
SCR = 0.6
Y_L=1
Z_S=1
Y_S=0
Z_S=0
Y_S^A=-0.4
Z_S^A=-2.5
Y = 0.6
Y_L + Y_S^A=0.6
Z=1.66
Y = 1/Z
Y = (1/Z_S)/(1 + 1/Z_S)

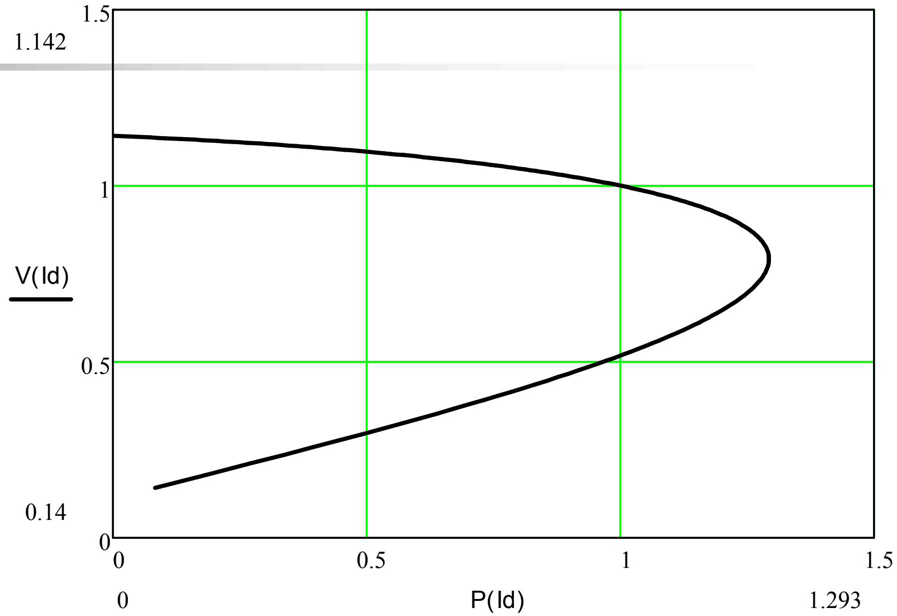

1.5
1.407
1
P(Id)
V(Id)
0.5
0
0
0
0.5
1
1.5
0
I(Id)
1.333

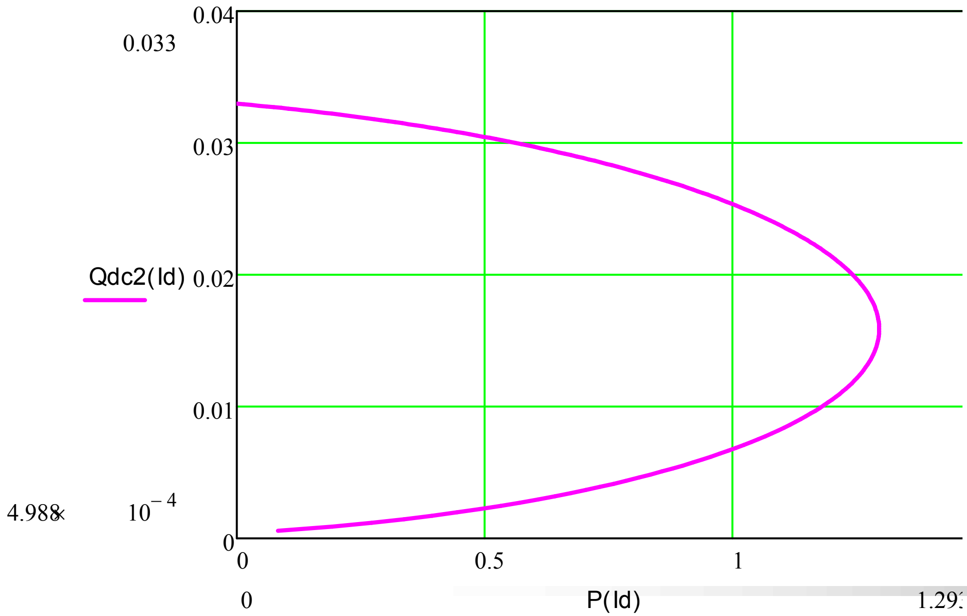

1.6
1.407
1.4
1.2
V(Id)
1
0.8
0.6
0.416
0.4
0
0.5
1
1.5
0
P(Id)
1


0.03
0.025
0.02
Qdc2(Id)
0.01
10^{-3}
0
0
0.5
1
0
P(Id)

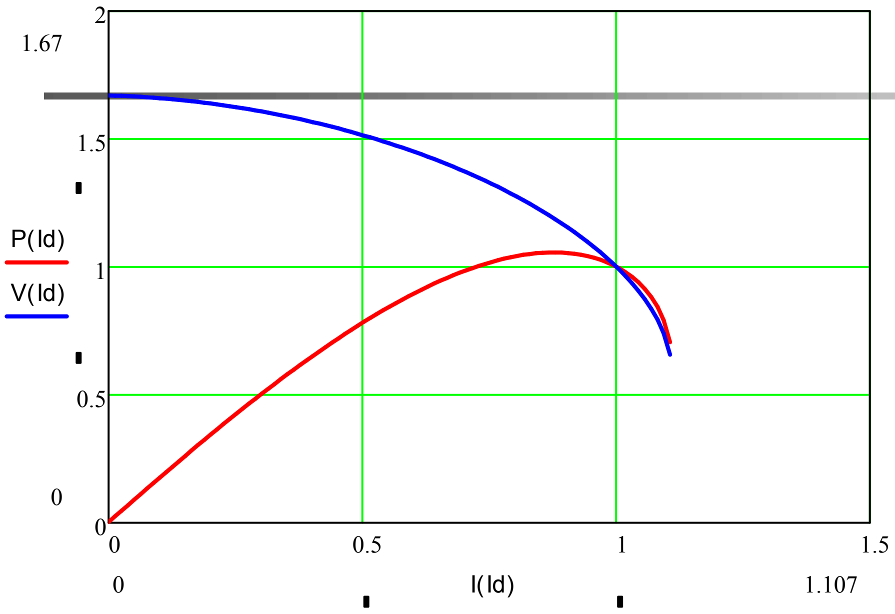
SCR=2


1.5
1.293
1
P(Id)
V(Id)
0.5
0
0
0
1
2
3
0
I(Id)
2.603

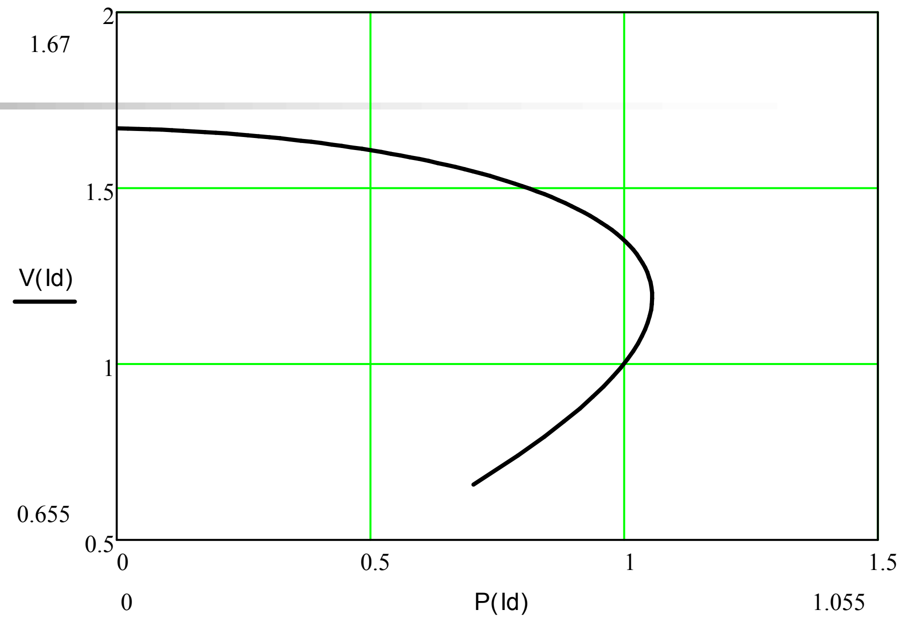

1.5
1.142
1
V(Id)
0.5
0.14
0
0
0.5
1
1.5
0
P(Id)
1.293


0.04
0.033
0.03
Qdc2(Id)
0.02
0.01
4.988 × 10⁻⁴
0
0
0.5
1
0
P(Id)
1.293

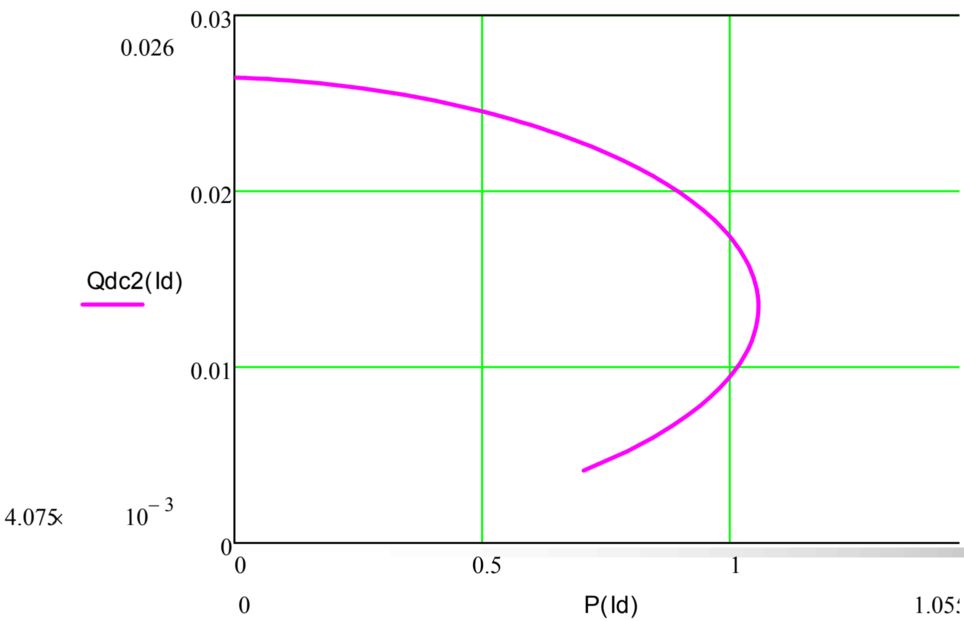
SCR=4



1.67
2
1.5
1
0.5
0
P(Id)
V(Id)
0
0.5
1
1.5
0
I(Id)
1.107

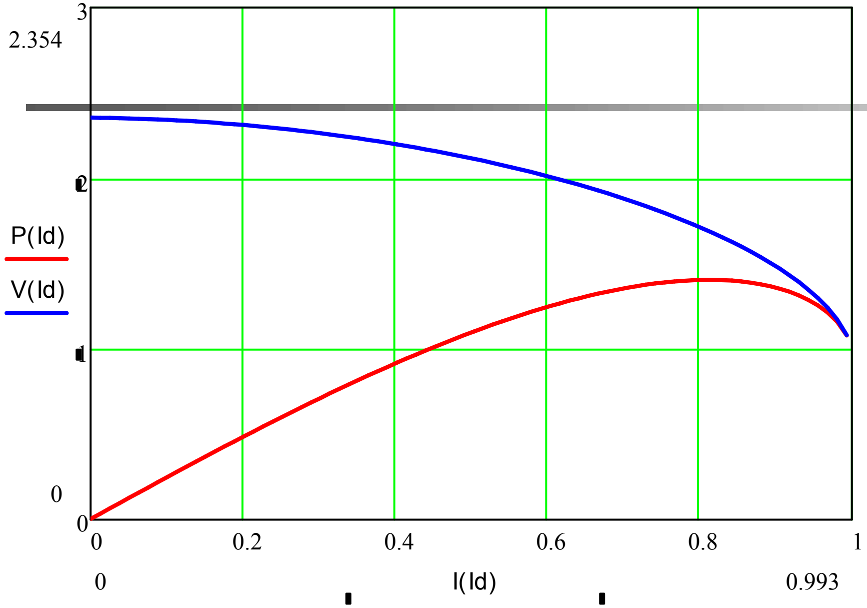


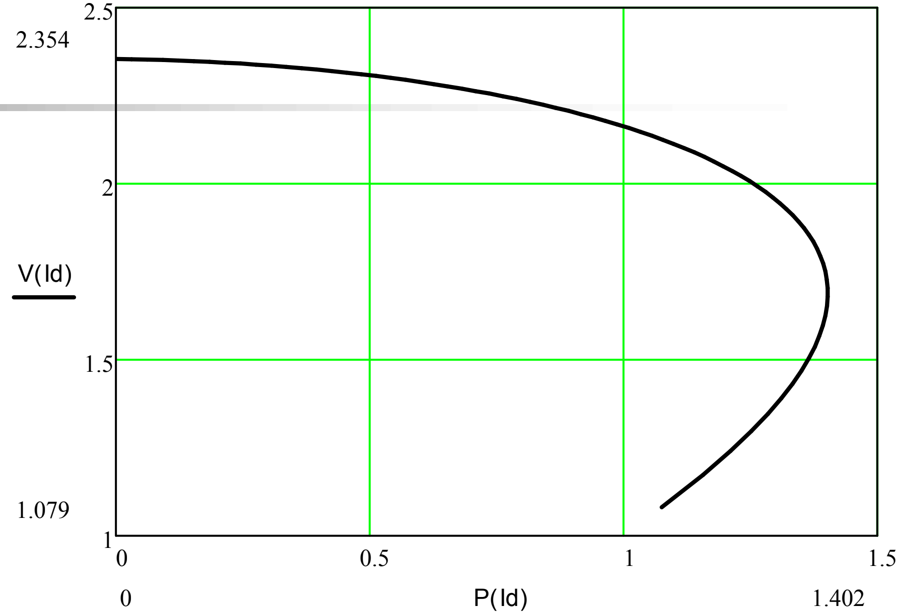
1.67
2
1.5
1
0.655
0.5
V(Id)
0
0.5
1
1.5
0
P(Id)
1.055

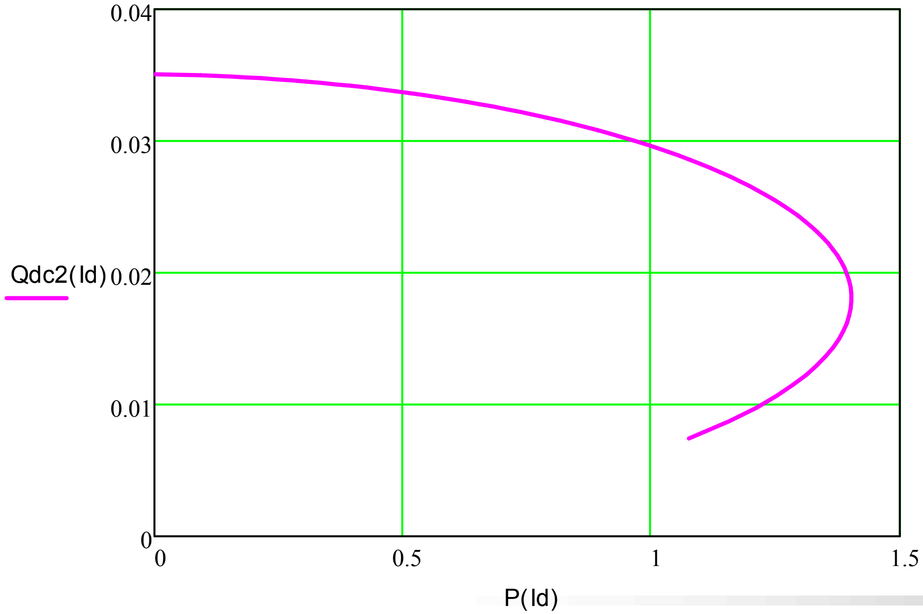


0.03
0.026
0.02
0.01
0
Qdc2(Id)
4.075×
10⁻³
0
0.5
1
0
P(Id)
1.05

SCR=1.5



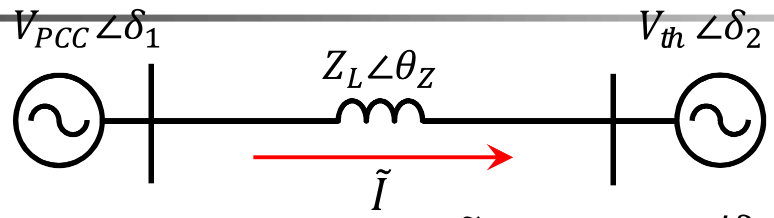




SCR=1

II. Grid Forming

Origin of Grid Forming Inverter



$$\begin{cases} \delta_1 = \theta_1 + \omega_1 t \\ \delta_2 = \theta_2 + \omega_2 t \\ \varphi^* = (\delta_1 - \delta_2) \end{cases}$$

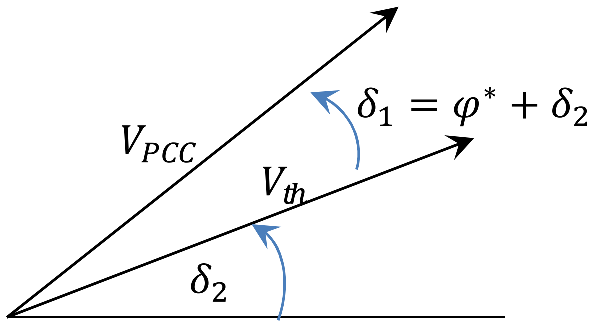
$$\tilde{V}_{PCC} = V_{PCC} \cdot e^{j\delta_1} \qquad \tilde{V}_{th} = V_{th} \cdot e^{j\delta_2}$$

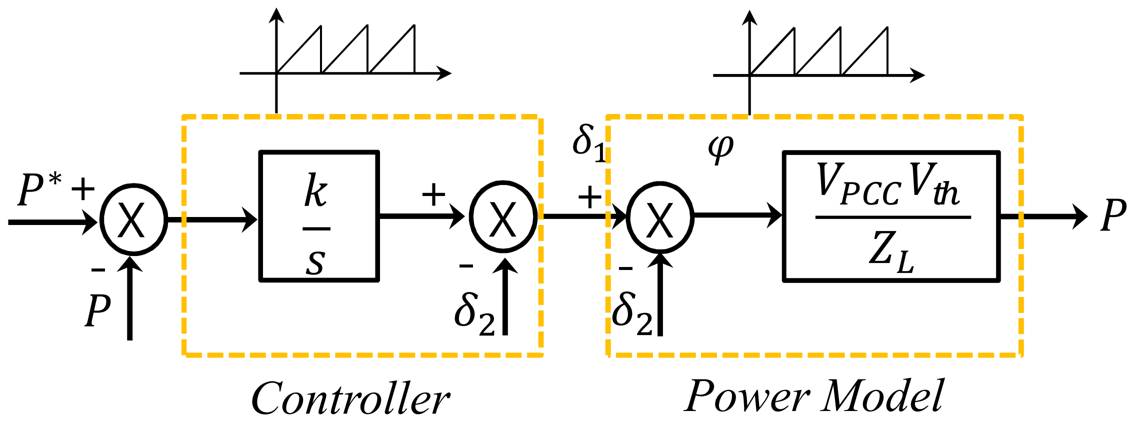
$$\tilde{I} = I \cdot e^{j(\delta_1 - \delta_2)}$$

$$P \approx \frac{V_{PCC} V_{th}}{Z_L} \cdot (\delta_1 - \delta_2)$$

$$\boldsymbol{(\delta_1 - \delta_2) = \varphi^* \approx P^* \frac{Z_L}{V_{PCC} V_{th}}}$$

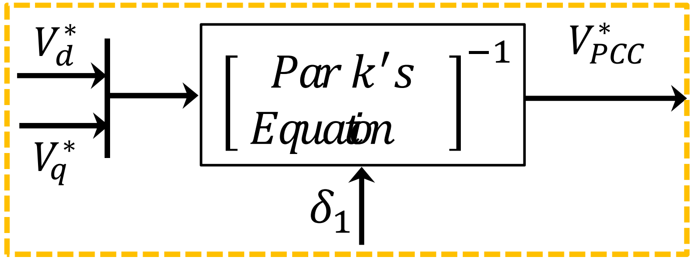
Origin of GFM



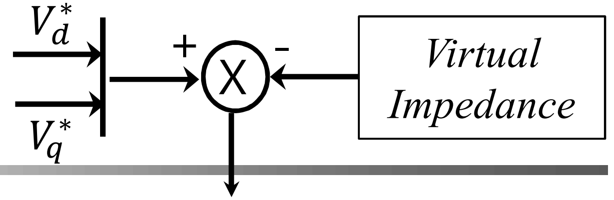


To maintain the angle constant, using PLL

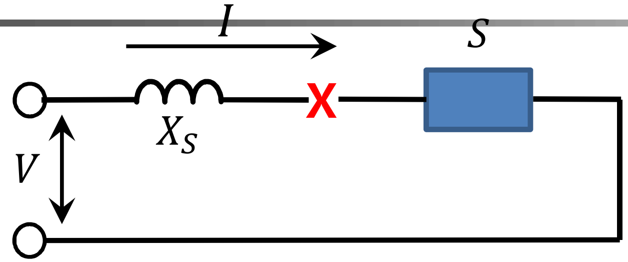
Closed loop is needed, to guarantee P = P^*



$\boldsymbol{X + X_{virtual}}$



X/R Ratio and Power Equation



$$P + jQ = V_1 I^* = V_1\left[\frac{V_1 - V_2}{Z_s}\right]^*$$

$$= V_1\left[\frac{V_1 - V_2 e^{j\delta}}{Z_s e^{-j\theta}}\right]$$

$$= \frac{V_1^2}{Z_s} e^{j\theta} - \frac{V_1 V_2}{Z_s} e^{j(\theta+\delta)}$$

$$P = \frac{V_1^2}{Z_s}\cos\theta - \frac{V_1 V_2}{Z_s}\cos(\theta + \delta)$$

$$Q = \frac{V_1^2}{Z_s}\sin\theta - \frac{V_1 V_2}{Z_s}\sin(\theta + \delta)$$

$$Z = R + jX$$

$$P = \frac{V_1}{(R^2 + X^2)}[R(V_1 - V_2\cos\delta) + XV_2\sin\delta]$$

$$Q = \frac{V_1}{(R^2 + X^2)}[-R(V_2\sin\delta) + X(V_1 - V_2\cos\delta)]$$

$R = 0 \quad Z = jX \ : \ \delta = P, V = Q$

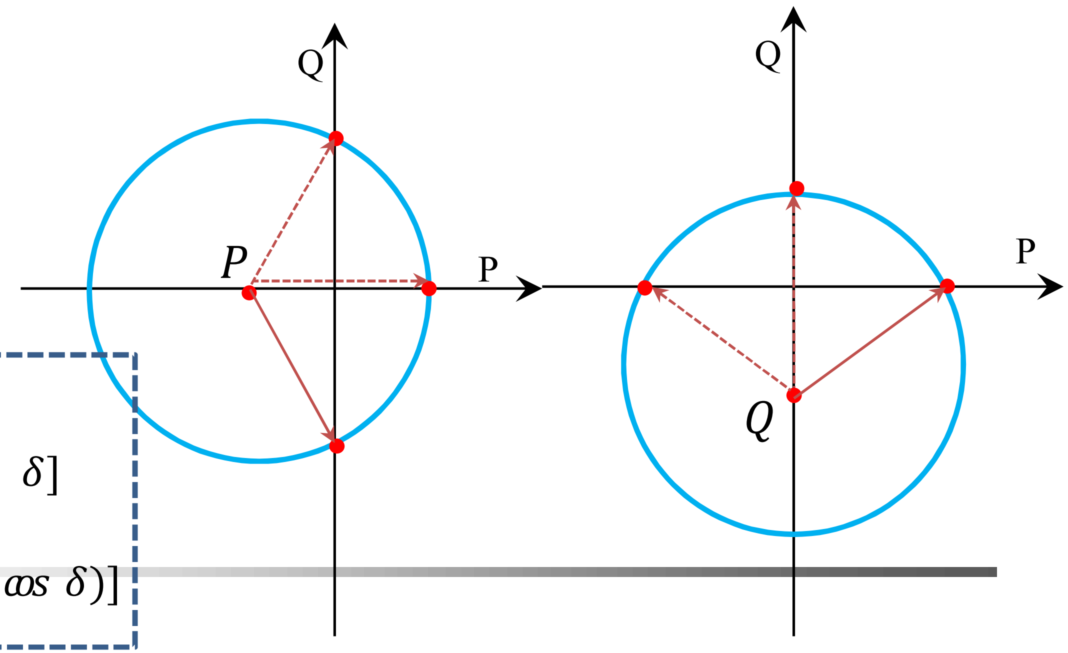
$$P = \frac{V_1}{X^2}[XV_2\sin\delta] = \frac{V_1 V_2}{X}\sin\delta$$

$$Q = \frac{V_1}{X^2}[X(V_1 - V_2\cos\delta)] = \frac{V_1^2 - V_1 V_2\cos\delta}{X}$$

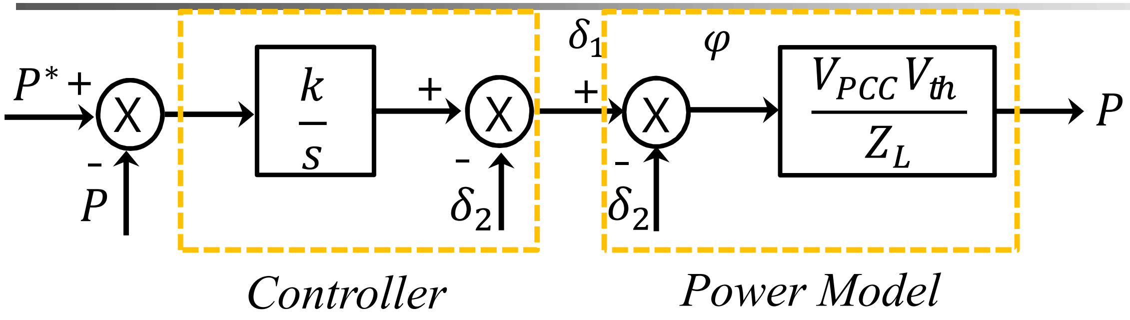
$X = 0 \quad Z = R : \ P = V, Q = f$

$$P = \frac{V_1}{R^2}[R(V_1 - V_2\cos\delta)] = \frac{V_1^2 - V_1 V_2\cos\delta}{R}$$

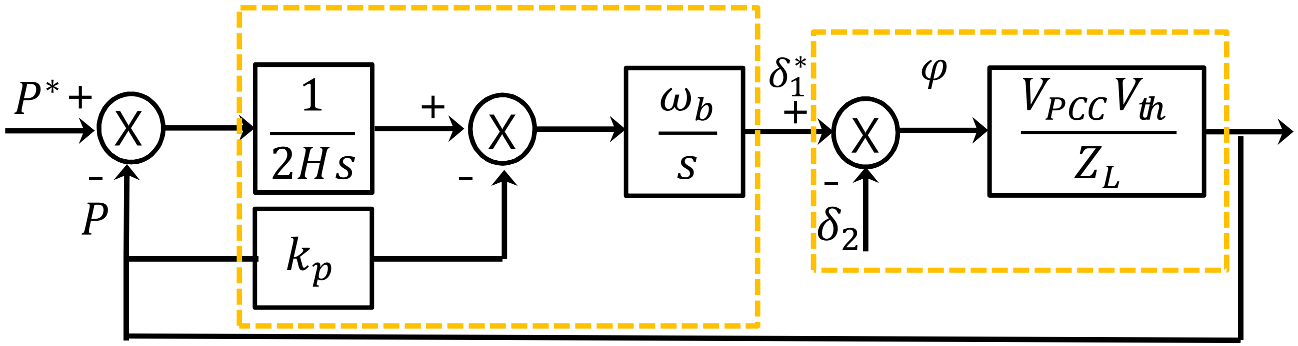
$$Q = \frac{V_1}{R^2}[-R(V_2\sin\delta)] = \frac{-V_1 V_2\sin\delta}{R}$$



Origin of Grid Forming Inverter



$$\Delta P_{Mech} = 2H\frac{d\omega}{dt} = \frac{2H}{\omega_b}\frac{d^2\delta}{dt^2}$$



$$P = \frac{1}{\frac{2HZ_L}{\omega_b}s^2 + 2Hk_p s + 1}P^* -= \frac{\frac{2H}{\omega_b}s^2}{\frac{2HZ_L}{\omega_b}s^2 + 2Hk_p s + 1}$$

$$P_c(s) = \frac{2HZ_L}{\omega_b}s^2 + 2Hk_p s + 1 = s^2 + 2\xi\omega_n + \omega_n^2$$

$$t_r(5\%) \approx \frac{3}{\omega_n} = 3\sqrt{\frac{2H(X_C + X_g)}{\omega_b}} = 3\sqrt{\frac{2HZ_L}{\omega_b}}$$

$$\begin{cases} \omega_n = \sqrt{\frac{\omega_b}{2HZ_L}} \\ \xi = k_p\sqrt{\frac{H\omega_b}{2X_C}} \\ k_p = \xi\sqrt{\frac{2X_C}{H\omega_b}} \end{cases}$$

Frequency strength

Grid Following

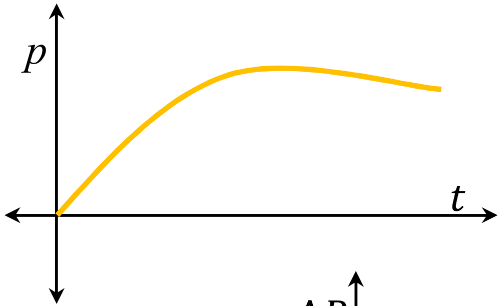
$\boldsymbol{\omega \rightarrow p}$

$$S = \sqrt{p^2 + q^2}$$

$$p = p^* - d\omega - H\frac{d\omega}{dt}$$

$$q = q^*$$

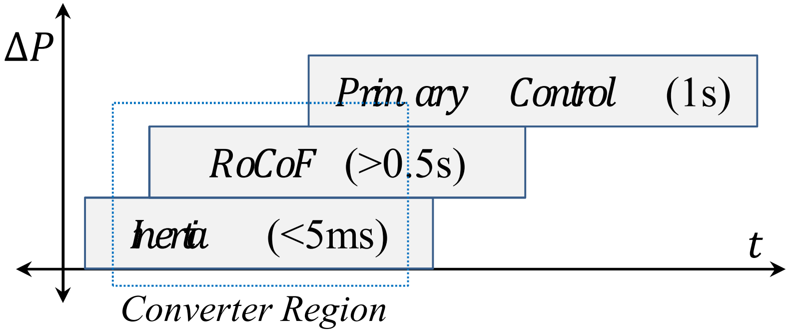
$$\omega(t) = \frac{1}{H}\int p_{DC}(t) - p_{ac}(t)$$



$$\boldsymbol{H}\frac{d\omega}{dt} = -D\omega - P_e + P_m$$

$$\boldsymbol{\tau_s}\frac{dP_m}{dt} = -P_m + P_m^* - K(\omega_0 - \omega_r)$$

$$\begin{cases} \text{Steam} & : 7\sim 10s \\ \text{Wind turbine} & : 0.1\sim 0.3s \\ \text{Battery} & : 0.05s \end{cases}$$



Grid Forming

$\boldsymbol{p \rightarrow \omega}$

$$S = \sqrt{p^2 + q^2}$$

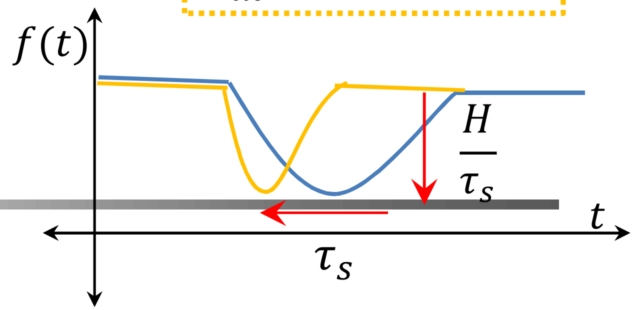
$$H_{virtual}\frac{d\omega}{dt} = -d\omega - p^* - p$$

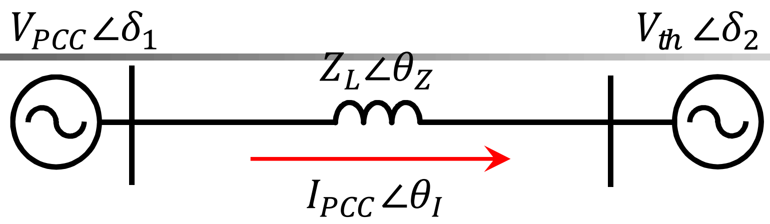
$$V = V^* + m_q(q^* - q)$$

$$V_{DC}(t) = \frac{1}{C_{DC}v_{DC}^*}\int p_{DC}(t) - p_{ac}(t)$$

$$\frac{\boldsymbol{H}}{\boldsymbol{\tau_s}}\frac{d\omega}{dt} = -P_e + P_m$$

$$\frac{dP_m}{dt} = -P_m + K\omega_s$$





$$V_{th}\angle\delta_2 = V_{PCC}\angle\delta_1 - Z_L\angle\theta_Z \cdot I_{PCC}\angle\theta_I$$

$$\delta = \delta_1 - \delta_2$$

$$S = P_{load} + jQ_{load}$$

$$S = V_2 I^*$$

$$\boldsymbol{I_{PCC}\angle\theta_I} = \frac{V_{PCC}\angle\delta - \boldsymbol{V_{th}}\angle 0}{\boldsymbol{Z_L\angle\theta_Z}}$$

$$\boldsymbol{I_{PCC}\angle\theta_I} = \frac{S}{V_{th}} = \frac{\boldsymbol{P_{load} - jQ_{load}}}{V_{th}\angle 0}$$

$$\boldsymbol{S_{PCC}\angle\delta_S = V_{PCC}\angle\delta \cdot I_{PCC}\angle\theta_I}$$

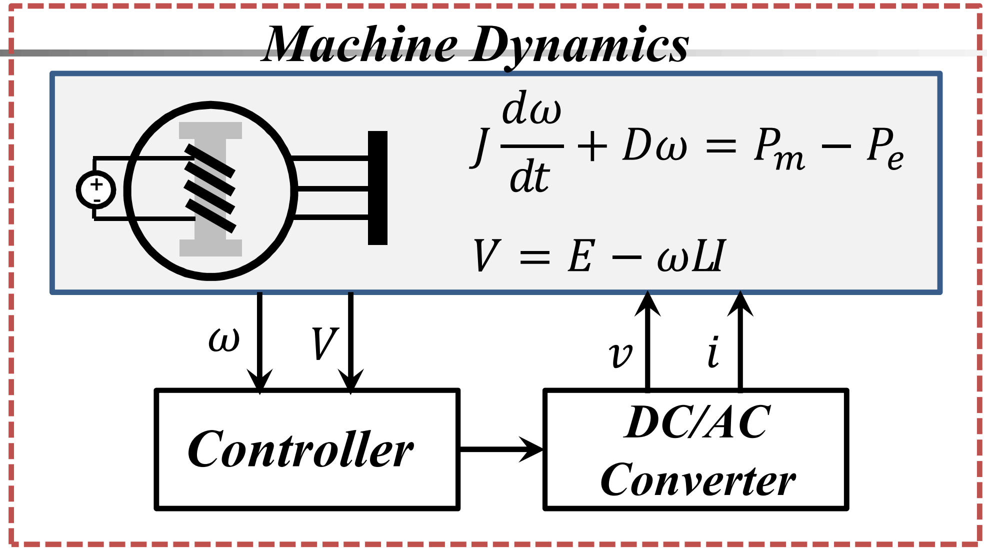
$$\boldsymbol{S_{PCC}} = \begin{cases} \text{Phase Shift} \\ (\delta_1) \\ \text{Magnitude Shift} \\ (S_{PCC} \text{ or } V_{PCC} \text{ or } I_{PCC}) \end{cases}$$

$$\boldsymbol{V_{th}} = \begin{cases} \text{Sag or Swell} \\ \text{1 Phase Fault} \\ \text{3 phase fault} \end{cases}$$

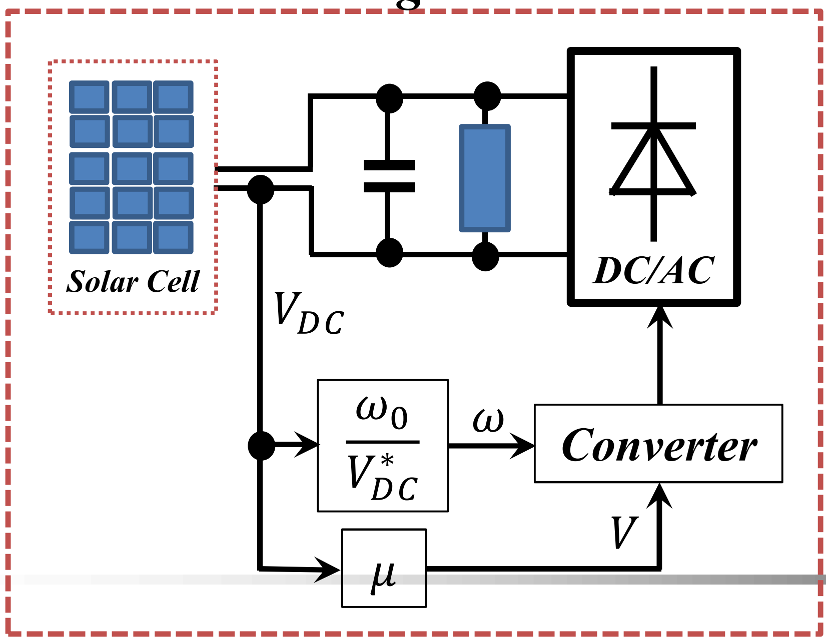
$$\boldsymbol{Z_L} = \begin{cases} |\boldsymbol{Z_L}| \\ (\text{Change Magnitude}) \\ jX \\ R + jX \end{cases}$$

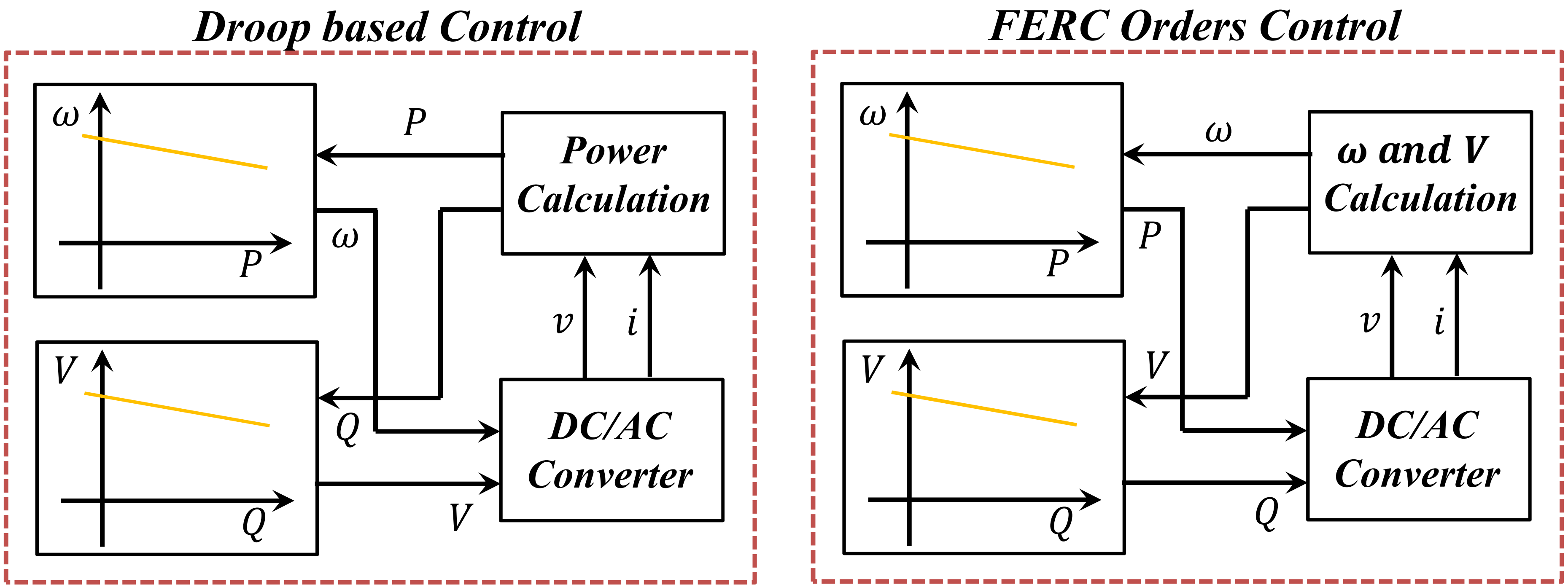
$$\boldsymbol{S_{PCC}} = \begin{cases} |\boldsymbol{S_{PCC}}| \\ (\text{Change Magnitude}) \\ P + jQ \text{ or } P \text{ or } jQ \\ \Delta P \text{ or } \Delta jQ \end{cases}$$

Virtual Synchronous Machine

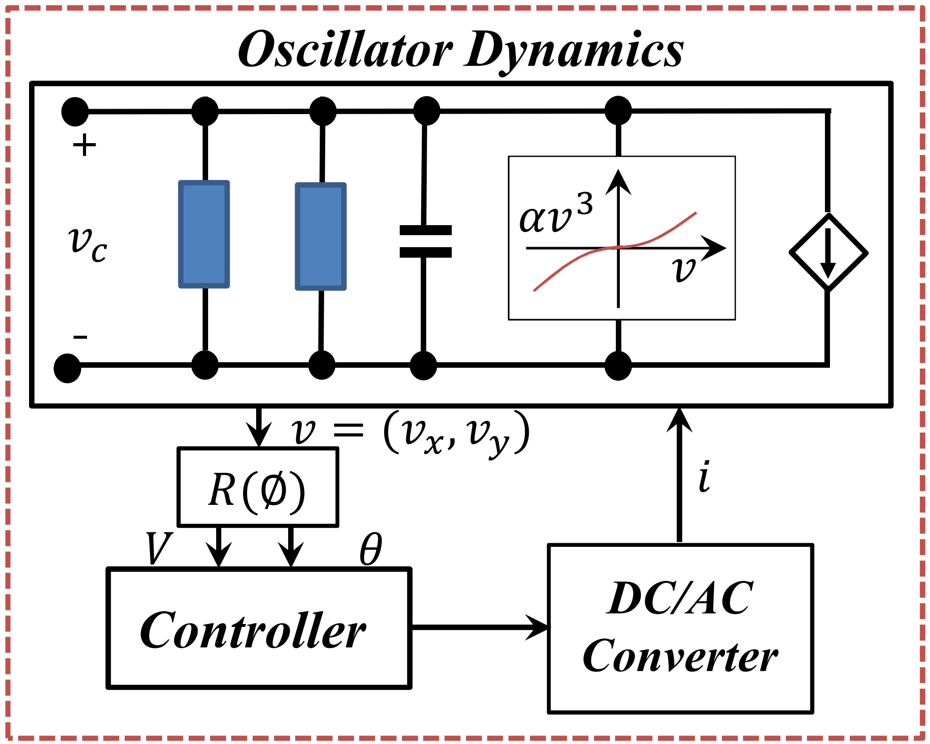


Matching Control

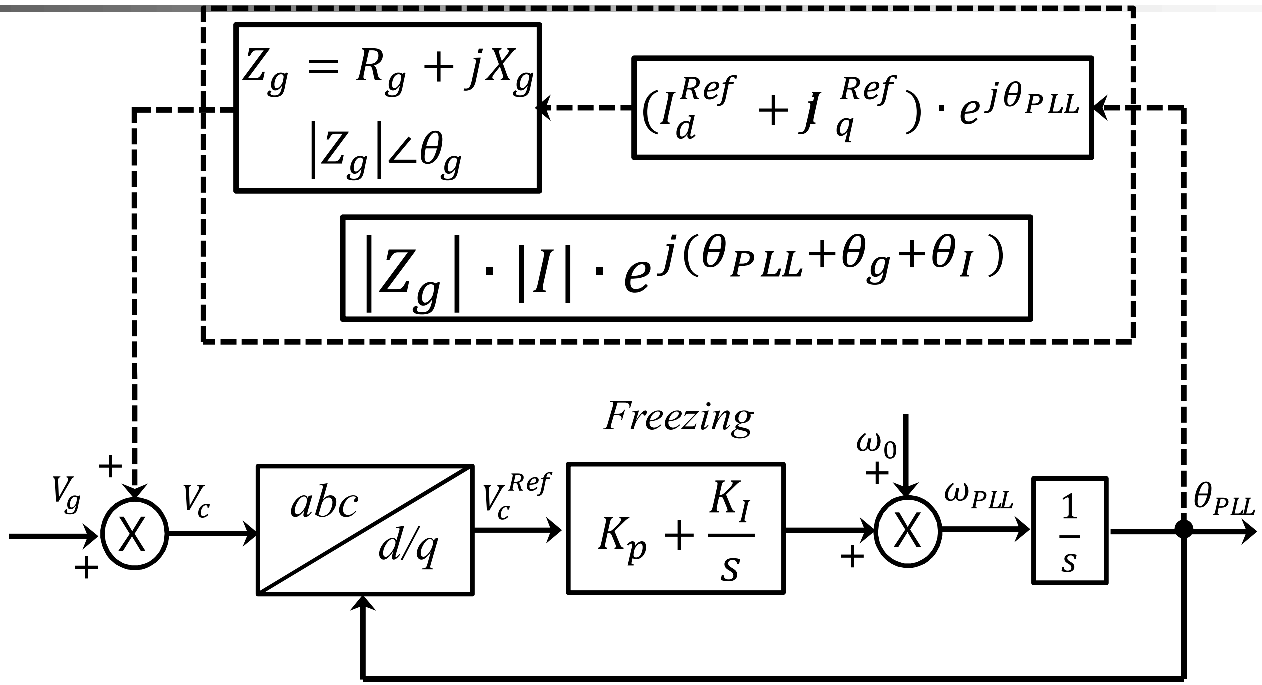



Droop based Control
ω
P
V
Q
P
ω
Q
V
Power Calculation
DC/AC Converter
v
i
FERC Orders Control
ω
P
V
Q
ω
P
V
Q
ω and V Calculation
DC/AC Converter
v
i

Virtual Oscillator Control



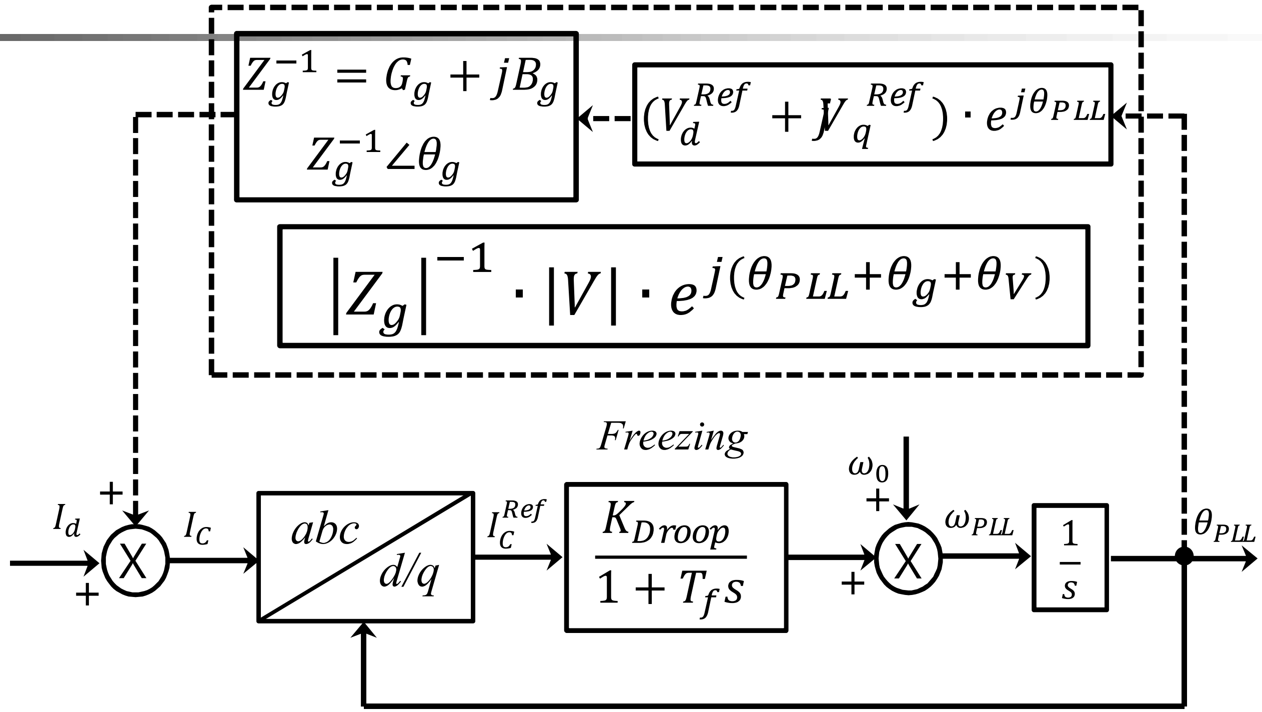
Grid Following Inverter



$$\theta_{PLL} = \begin{cases} \theta_g = \tan^{-1}\left(\frac{X_g}{R_g}\right) \\ \theta_I = \tan^{-1}\left(\frac{I_q}{I_d}\right) \\ |Z_g| = 1/SCR \\ |I| = \text{Converter Rating} \\ |V_g| = \text{Grid Voltage} \end{cases}$$

$$V_g = \begin{cases} \text{Sag} \\ \text{Swell} \\ \text{SCR Variation} \\ \text{Fault} \\ \text{Load Rejection} \end{cases} \qquad \text{Freezing} = \begin{cases} \text{Fault} \\ \text{Inertia} \rightarrow \infty \\ K_I \rightarrow \text{Inertia} \\ K_p \text{ and } K_I \rightarrow \\ \text{Damping} \end{cases}$$

Grid Forming Inverter

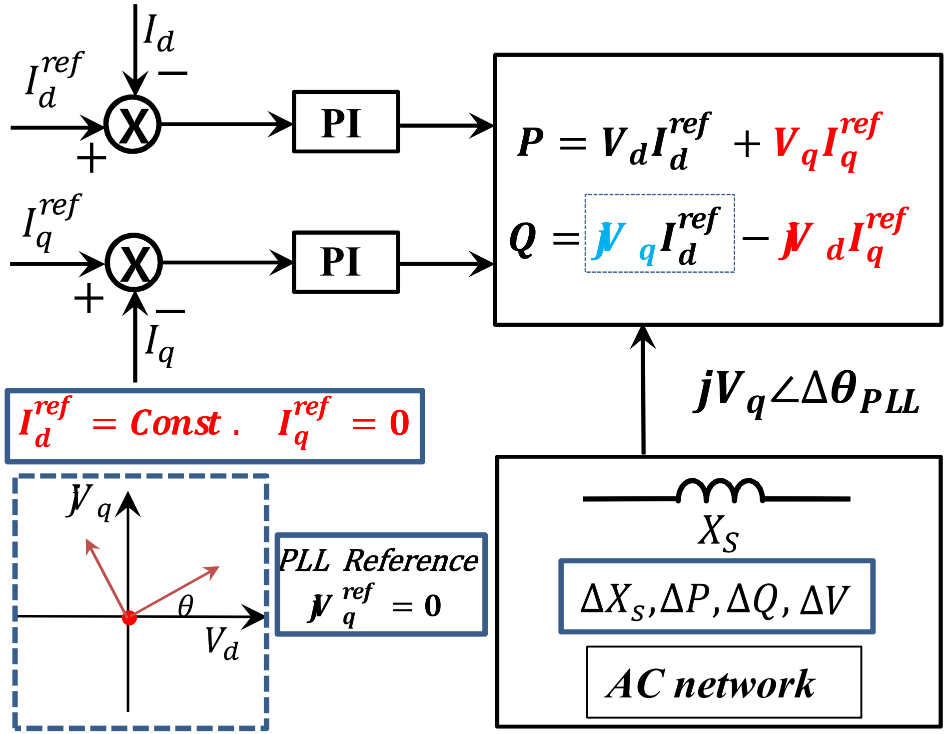


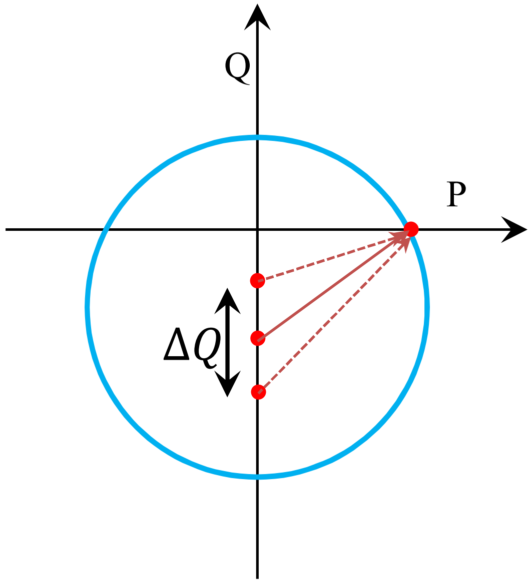
$$\theta_{PLL} = \begin{cases} \theta_g = \tan^{-1}\left(\frac{B_g}{G_g}\right) \\ \theta_V = \tan^{-1}\left(\frac{V_q}{V_d}\right) \\ \left|Z_g\right| = 1/SCR \\ |V| = Converter\ Rating \\ |I_d| = Converter\ current \end{cases}$$

$$\begin{cases} K_{Droop} : \frac{P_{converter}}{\sum P_{total}} \\ T_f : Time\ constant \rightarrow Transient\ Stability \end{cases}$$

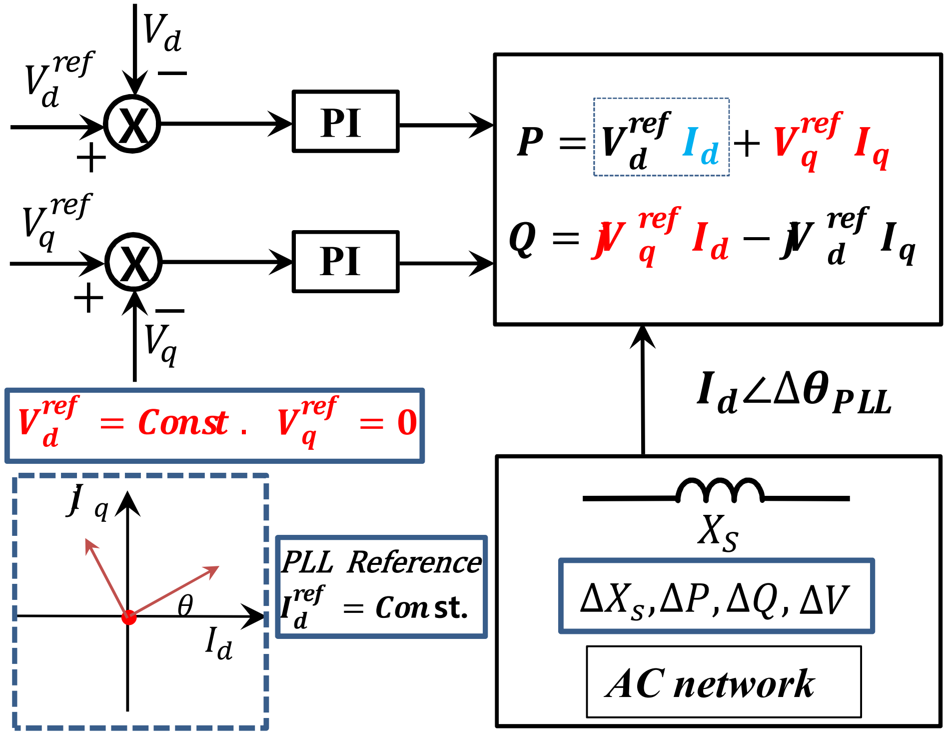
$$I_d = \begin{cases} SCR\ Variation \\ Fault \\ Load\ Rejection \end{cases}$$

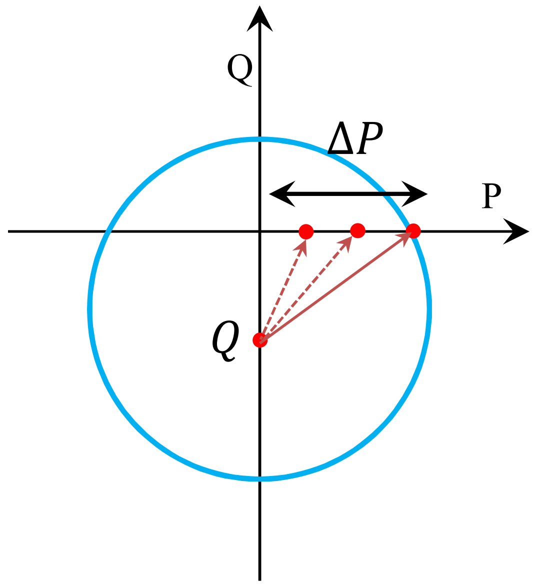
$$Freezing = \begin{cases} Fault \\ K_{Droop} \rightarrow Inertia \\ \rightarrow \infty \\ Damping \end{cases}$$





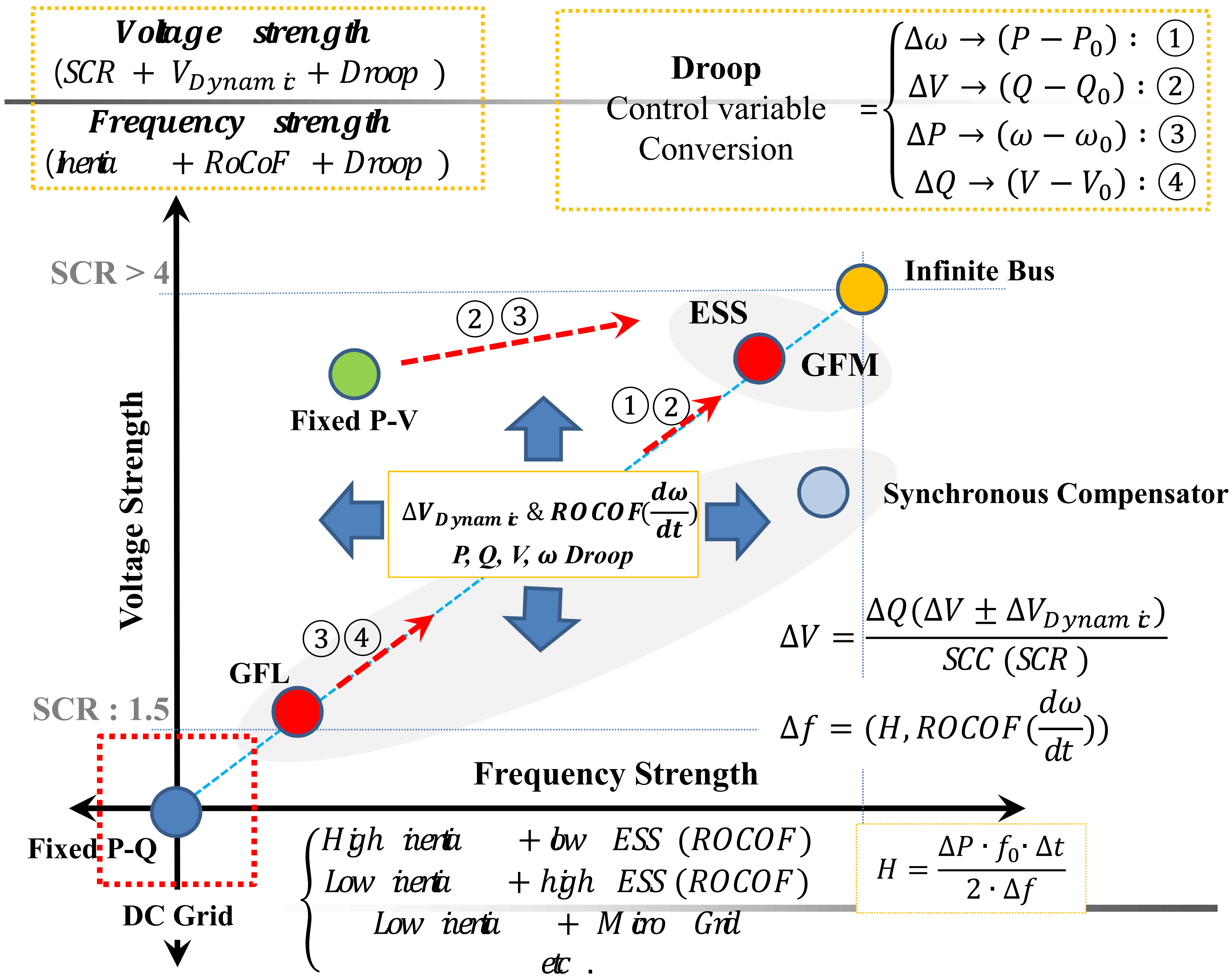
Grid-Following (Voltage-Following)
Current Forming (Active Power Forming)





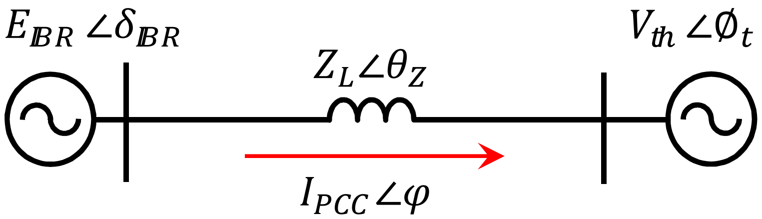
Grid-Forming (Voltage-Forming)

Current Following (Reactive Power Forming)



Voltage strength
(SCR + V_Dynamic + Droop)
Frequency strength
(Inertia + RoCoF + Droop)
Droop
Control variable
Conversion
Δω → (P − P0) : ①
ΔV → (Q − Q0) : ②
ΔP → (ω − ω0) : ③
ΔQ → (V − V0) : ④
SCR > 4
Infinite Bus
ESS
GFM
Fixed P-V
①②
②③
Synchronous Compensator
Voltage Strength
ΔV_Dynamic & ROCOF(dω/dt)
P, Q, V, ω Droop
ΔV = ΔQ(ΔV ± ΔV_Dynamic) / SCC (SCR)
③④
GFL
SCR : 1.5
Δf = (H, ROCOF(dω/dt))
Frequency Strength
Fixed P-Q
DC Grid
High inertia + low ESS (ROCOF)
Low inertia + high ESS (ROCOF)
Low inertia + Micro Grid
etc.
H = ΔP · f0 · Δt / 2 · Δf

Grid Following Inverter



$$(\boldsymbol{I_{PCC}\angle\varphi})_{ref} = \boldsymbol{I_{PCC}\angle\varphi} \qquad \emptyset_{PLL} = \emptyset_t$$

AC network

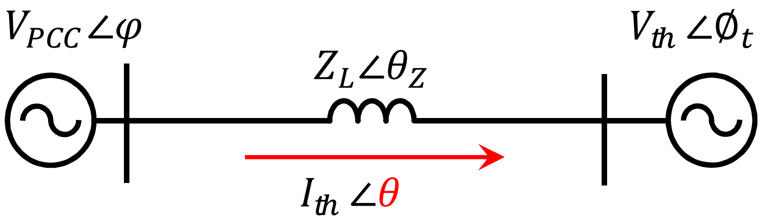
$$\boldsymbol{I_{PCC}\angle\varphi} = \frac{E_{IBR}\angle\delta_{IBR} - \boldsymbol{V_{th}}\angle\emptyset_t}{\boldsymbol{Z_L\angle\theta_Z}}$$

$$(\boldsymbol{I_{PCC}\angle\varphi})_{ref} = \frac{P_{ref} - j\boldsymbol{Q}_{ref}}{\boldsymbol{V_{th}}\angle - \emptyset_{PLL}}$$

Controller

$E_{IBR}\angle\delta_{IBR}$ must change rapidly when $V_{th}\angle\emptyset_t$ changes

Grid Forming Inverter



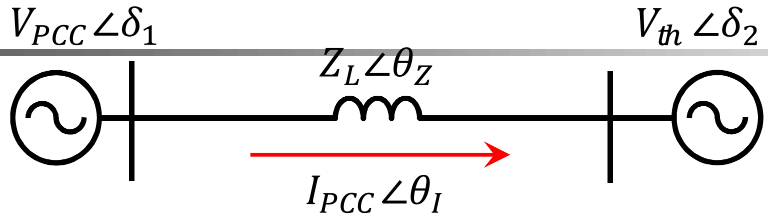
$$(\boldsymbol{V_{PCC}\angle\varphi})_{ref} = \boldsymbol{V_{PCC}\angle\varphi} \qquad \theta = \theta_{PLL}$$

AC network

$$\boldsymbol{V_{PCC}\angle\varphi} = \boldsymbol{Z_L\angle\theta_Z} \cdot \boldsymbol{I_{th}\angle\theta} + \boldsymbol{V_{th}}\angle\emptyset_t$$

$$(\boldsymbol{V_{PCC}\angle\varphi})_{ref} = \frac{P_{ref} - j\boldsymbol{Q}_{ref}}{\boldsymbol{I_{th}}\angle - \boldsymbol{\theta_{PLL}}}$$

Controller



$$V_{th}\angle\delta_2 = V_{PCC}\angle\delta_1 - Z_L\angle\theta_Z \cdot I_{PCC}\angle\theta_I$$

$$\delta = \delta_1 - \delta_2$$

$$S = P_{bad} + jQ_{bad}$$

$$S = V_2 I^*$$

$$\boldsymbol{I_{PCC}\angle\theta_I} = \frac{V_{PCC}\angle\delta - \boldsymbol{V_{th}}\angle 0}{\boldsymbol{Z_L\angle\theta_Z}}$$

$$\boldsymbol{I_{PCC}\angle\theta_I} = \frac{S}{V_{th}} = \frac{\boldsymbol{P_{bad} - jQ_{bad}}}{V_{th}\angle 0}$$

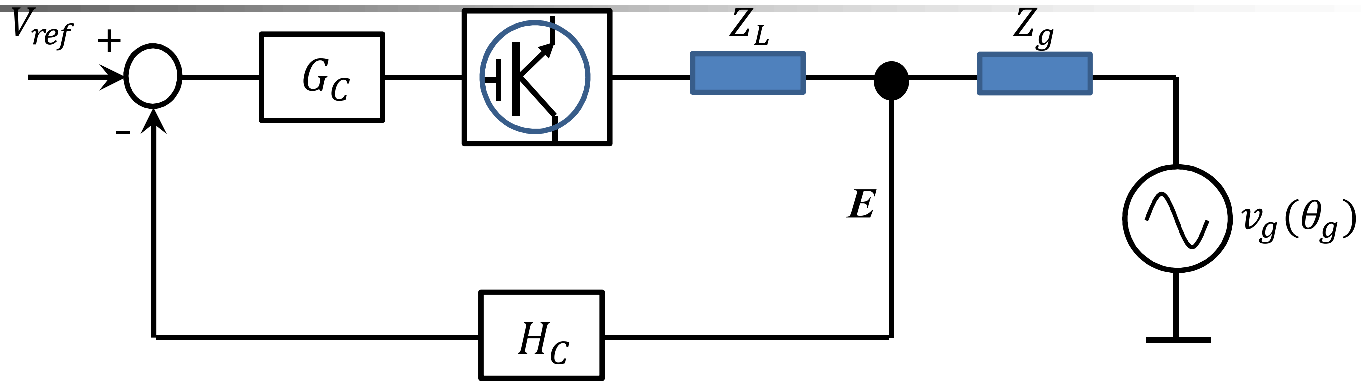
$$\boldsymbol{S_{PCC}\angle\delta_S = V_{PCC}\angle\delta \cdot I_{PCC}\angle\theta_I}$$

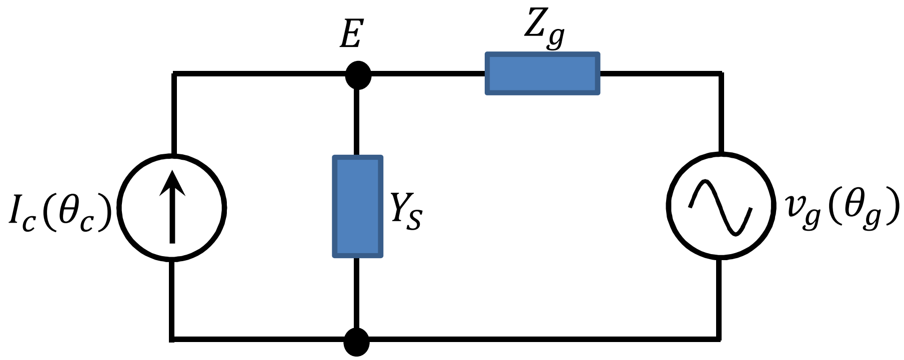
$$\boldsymbol{S_{PCC}} = \begin{cases} Phase\ \ hn\,t \\ (\delta_1) \\ M\,agntude\ \ hn\,t \\ (S_{PCC}\ or\ V_{PCC}\ or\ I_{PCC}) \end{cases}$$

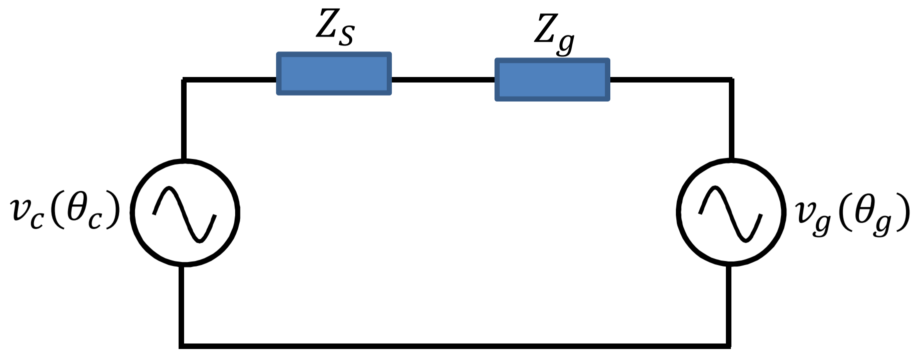
$$\boldsymbol{V_{th}} = \begin{cases} Sag\ \ or\ \ Swel \\ 1\ Phase\ \ Faut \\ 3\ phase\ \ faut \end{cases}$$

$$\boldsymbol{Z_L} = \begin{cases} |\boldsymbol{Z_L}| \\ (Change\ \ M\,agntude\ ) \\ jX \\ R + jX \end{cases}$$

$$\boldsymbol{S_{PCC}} = \begin{cases} |\boldsymbol{S_{PCC}}| \\ (Change\ \ M\,agntude\ ) \\ P + jQ\ \ or\ \ P\ or\ jQ \\ \Delta P\ or\ \Delta jQ \end{cases}$$







$$I_c(\theta_c) = \frac{G_c \cdot V_{ref}}{Z_L} \qquad Y_S(\theta_s) = \frac{1 + G_c \cdot H_c}{Z_L} \qquad v_c(\theta_c) = \frac{G_c \cdot V_{ref}}{1 + G_c \cdot H_c} \qquad Z_S(\theta_s) = \frac{Z_L}{1 + G_c \cdot H_c}$$

$$i = \frac{G_c(s)}{1 + Ys(s) \cdot \boldsymbol{Z_g(s)}} \cdot i_{ref} + \frac{Y(s)}{1 + Ys(s) \cdot \boldsymbol{Z_g(s)}} \cdot v_g$$

Extended SCR-Plus

$$SCR = \frac{SCL\ (Short\ \ Circuit\ \ Ratio\ \ (MVA))}{P_{DC}(DC\ power\ (MW))} = \frac{1}{Z_{ac}}$$

[Power ratio and Inverse impedance]

$$\boldsymbol{SCR_{plus}} = \frac{SCL\ (MVA) + \boldsymbol{Converter\ \ Reactive\ \ Power} + \boldsymbol{ESS\ (active\ \ control\ )}}{P_{DC}(DC\ power\ (MW))}$$

$$H_{DC} = \frac{H_1 \cdot MW_1 + H_2 \cdot MW_2 + \boldsymbol{H_{ESS}} \cdot MW_3}{MW_1 + MW_2 + MW_3 + MW_{DC-base}\ (SCR\ )}$$

$$\boldsymbol{SCR_{inertia}} = \boldsymbol{SCR_{base}} - \boldsymbol{2 \cdot H_{DC} \cdot \frac{\Delta f}{\Delta t}\Big|_{\frac{\Delta f}{\Delta t}=0\ :\ Steady\ -State}^{Negative\ \ value\ \ :\ Stable}}$$

Voltage stability, Power Ratio
MVA, dV/dI

Inertia, MVA, df/dt

Passive component :

Gen, TL, S.C,

Base HVDC

Active component :

GFM(HVDC, FACTs, ESS),

GFL(HVDC, FACTs, ESS),

CSC FACTs, ***ESS***

GFM

$$SCR_{plus} = \frac{MVA \pm MVA\angle\theta_C}{MW \pm MW\angle\theta_C}$$

GFL

Control Tuning

$$SCR_{inertia} = SCR_{base} - 2 \cdot H_{DC} \cdot \frac{\Delta f}{\Delta t} \begin{cases} Negative\ value : Stable \\ \frac{\Delta f}{\Delta t} = 0 : Steady-State \end{cases}$$

$$H_{DC} = \frac{H_1 \cdot MW_1 + H_2 \cdot MW_2 + H_{ESS} \cdot MW_3}{MW_1 + MW_2 + MW_3 + MW_{DC-base}(SCR)}$$

$$\boldsymbol{SCR^{+}} \quad \boldsymbol{SCR_{plus}} = \begin{cases} *\ Conventional\ \ SCR \\ *\ CSC\ \ HVDC\ based \\ *\ Grid\ \ Forming \\ *\ Grid\ \ Following \\ *\ CSC\ \ FACTs \\ *\ VSC\ \ FACTs \\ *\ Inertia\ \ required \\ *\ \boldsymbol{Controllable\ \ SCR} \end{cases}$$

$$\boldsymbol{EnSCR_{plus}} \quad \boldsymbol{EnSCR^{+}} = \begin{cases} *\ Conventional\ \ SCR \\ *\ CSC\ \ HVDC\ based \\ *\ Grid\ \ Forming \\ *\ Grid\ \ Following \\ *\ CSC\ \ FACTs \\ *\ VSC\ \ FACTs \\ *\ Inertia\ \ required \\ *\ \boldsymbol{Controllable\ \ SCR} \end{cases} + \begin{cases} *\ Controller\ \ stability \\ *\ Super\ \ Resonance \\ *\ Sub\ \ Resonance \\ *\ PLL\ \ Oscillation \\ *\ AC/DC\ \ Interaction \\ *\ Delay\ \ instability \end{cases}$$

$Extended\ \ SCR_{plus}$ $\qquad$ $Extended\ \ SCR^{+}$

II. Proposed PLL

Thank You !!